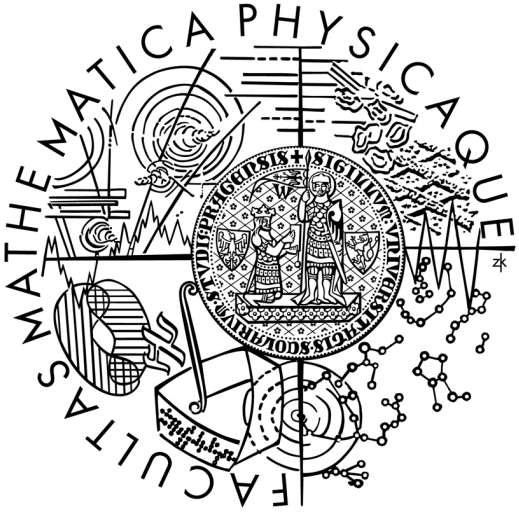
Charles University in Prague
Faculty of Mathematics and Physics

24th Annual Student Conference

Week of Doctoral Students 2015



Book of Abstracts

of the

Week of Doctoral Students of the School of Mathematics 2015 June 8, 2015

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http://www.karlin.mff.cuni.cz/~rokyta/WDS-M/2015/
http://www.mff.cuni.cz/veda/konference/wds/

Vytisklo Reprostředisko MFF UK Praha

WDS - M 2015, 8.6.2015, K1

Conference schedule

8 : 55 - 9 : 00	**Opening**						
9 : 00 - 9 : 25	1	Mgr.	Břetislav	Skovajsa	4M3	Generalized Ordinary Differential Equations in Metric Spaces
9 : 25 - 9 : 50	2	Mgr.	Vít	Musil	4M3	Optimal Orlicz domains in Sobolev embeddings
9 : 50 - 10 : 15	3	Mgr.	Martin	Franců	4M3	Higher-order Sobolev-type imbeddings on Carnot-Carathéodory spaces
10 : 15 - 10 : 40	4	Mgr.	Martin	Křepela	4M3	TBA
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10 : 50 - 11 : 15	5	Mgr.	Tereza	Bártlová	4M8	Interactive Science Centers
11 : 15 - 11 : 40	6	Mgr.	Tereza	Kovářová	4M8	Tiling Proofs of Combinatorial Identities Involving Fibonacci, Lucas and Pell Numbers
11 : 40 - 12 : 05	7	Mgr.	Michal	Zamboj	4M8	Synthetic approach to Chasles theorem for timelike ruled surface
12 : 05 - 13 : 35	**Break**					
13 : 35 - 14 : 00	8	Mgr.	Jan	Horáček	4M1	Network coding
14 : 00 - 14 : 25	9	Mgr.	Adam	Bartoš	4M2	Maximal connected topologies
14 : 25 - 14 : 50	10	Mgr.	Petr	Zima	4M2	Prolongation of Killing equations
14 : 50 - 15 : 00	**Break**					
15 : 00 - 15 : 25	11	Mgr.	Filip	Roskovec	4M6	Gauss-Radau polynomial reconstruction of discontinuous
15 : 25 - 15 : 50	12	Mgr.	Petr	Vágner	4F11	Optimization of fuel cells
15 : 50 - 16 : 00	**Break**					
16 : 00 - 16 : 25	13	Mgr.	Jakub	Večeřa	4M9	Central limit theorem for Gibbsian U-statistics of facet processes
16 : 25 - 16 : 50	14	Mgr.	Petr	Čoupek	4M4	Cylindrical Volterra Processes and Stochastic Evolution Equations
16 : 50 - 17 : 15	15	Mgr.	Karel	Kadlec	4M4	Ergodic Control for Infinite Dimensional Lévy Processes
17 : 15 - 17 : 40	16	Mgr.	David	Coufal	4M4	Kernel methods in particle filter

Preface

In the beginning of 2014, the Management of the Faculty of Mathematics and Physics decided that the traditional conference of PhD students called the WDS (Week of Doctoral Students) would not be organized as an activity of the entire faculty. Instead, the decision as to whether to organize the conference or not was left to the respective Schools (of Computer Science, of Mathematics, and of Physics).

Already for the second year since then the School of Mathematics organizes this WDS-M (Week of Doctoral Students of the School of Mathematics, `http://www.karlin.mff.cuni.cz/~rokyta/WDS-M/2015/`), this time as a one-day conference, in the framework, and as a continuation of, the (24th) WDS of the Faculty of Mathematics and Physics (`http://www.mff.cuni.cz/veda/konference/wds/`).

This year 16 students have registered as active participants to the conference. We believe that this event, which takes place in the "mathematical" Karlín building of the faculty, will attract the attention of the students but also of the broad mathematical audience. We thus encourage all of those interested in the scientific activities of our doctoral students to attend this meeting.

Prague, June 8, 2015

doc. RNDr. Mirko Rokyta, CSc.
Vice-Dean for Mathematics
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Contents

Network coding

Mgr. Jan Horáček

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Obor studia: 4M1 – Algebra, teorie čísel a matematická logika
Ročník: 1.
Školitel: prof. RNDr. Aleš Drápal, CSc., DSc.

Abstract

Network coding is a new powerful technique used in networking and it has been studied extensively as a part of coding theory. Network coding, as opposed to traditional error-correcting codes for communication through one channel, assumes a richer infrastructure – a network. We describe main principles behind network coding and explore its algebraic background. A random linear network coding scheme, which is one of the most practical implementation of network coding, is presented.

Maximal connected topologies

Mgr. Adam Bartoš

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Ročník: 1.
Školitel: prof. RNDr. Petr Simon, DrSc.

Abstract

If $\mathscr{P}$ is a property of topological spaces, we say that a topological space $\langle X, \tau \rangle$ (or the corresponding topology τ) is *maximal* $\mathscr{P}$ if it has the property $\mathscr{P}$, but no strictly finer topology $\tau^* > \tau$ has the property $\mathscr{P}$. We are interested in the case where $\mathscr{P}$ means connectedness, i.e. in *maximal connected topologies*. There are examples of Hausdorff maximal connected spaces as well as examples of Hausdorff connected spaces having no finer topology that is maximal connected. The problem of existence of a regular maximal connected topology remains open.

We will summarize the history of the problem of existence of maximal connected spaces with prescribed separation axioms, present known facts, and illustrate them on several examples.

Prolongation of Killing equations

Mgr. Petr Zima

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Ročník: 1.
Školitel: doc. RNDr. Petr Somberg, Ph.D.

Abstract

Killing equations are certain systems of partial differential equations arising naturally in (pseudo-)Riemannian and *Spin*-geometry. The systems are always overdetermined and hence the existence of nontrivial solutions is a rare phenomenon. But once the solutions exist, they provide valuable information about geometry of the underlying manifold. In particular, the equations have a strong intrinsic relationship with curvature.

Effective tool for approaching the Killing equations is the so called prolongation procedure. It transforms the system of differential equations into a larger equivalent system of simpler nature. This transformation provides both theoretical and practical advantage. Firstly, it almost immediately yields an upper bound for the dimension of the space of solutions. Secondly, in some specific examples the transformed system can be directly integrated yielding an explicit formula for the solutions.

The Killing equations come in different types depending on the quantity which constitutes the domain of the equation. The most prominent example are the Killing vector fields. They directly correspond to infinitesimal symmetries of the underlying manifold. As such, they are widely known and used in the fields of (pseudo-)Riemannian geometry and also theoretical physics. In this talk I will focus on a more complicated type whose domain is a quantity called spinor-valued differential forms. To my best knowledge, systematic study of this case is original work.

Higher-order Sobolev-type imbeddings on Carnot-Carathéodory spaces

Mgr. Martin Franců

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Ročník: 3.
Školitel: prof. RNDr. Luboš Pick, CSc., DSc.

Abstract

Sufficient condition for higher-order Sobolev-type imbeddings on bounded domains of Carnot-Carathéodory spaces is established for class of rearrangement-invariant function spaces. The condition takes form of a one-dimensional inequality for suitable integral operators depending on the isoperimetric function relative to the Carnot-Carathéodory structure of the relevant sets. General results are then applied to particular Sobolev spaces build upon Lebesgue, Lorentz and Orlicz spaces on John domains in Heisenberg group. In the case of Heisenberg group and first-order Sobolev-type imbedding, the condition is shown to be necessary as well.

Iterating bilinear inequalities

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Abstract

Consider a bilinear operator constructed as a product of two ordinary Hardy operators. Then it is possible to obtain weighted inequalities involving this operator, by iteration and using the classical weighted Hardy inequalities. In this way, one gets various results in form of inequalities for nonnegative, decreasing functions or other classes. The idea applies easily to various Hardy-type bilinear or multilinear operators, but may as well be used to obtain estimates for convolution-type operators.

Optimal Orlicz domains in Sobolev embeddings

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Abstract

For a given Banach function space $Y(\Omega)$, we study the question whether there exists an optimal (i.e. largest) Orlicz space $L^A(\Omega)$ satisfying the embedding

$$W^m L^A(\Omega) \hookrightarrow Y(\Omega),$$

where Ω stands for a bounded domain in $\mathbb{R}^n$, $n \geq 2$, and $W^m L^A(\Omega)$ is an Orlicz-Sobolev space.

In the general setting, where in the place of $L^A(\Omega)$ there is a rearrangement-invariant (r.i.) Banach function space (i.e. space where functions having the same distribution function have the same norm), such questions were investigated using the method of reducing the Sobolev embeddings to the boundedness of an appropriate modification of the weighted Hardy operator. In the setting of r.i. spaces, the optimal domain and the optimal target spaces always exist and they are then explicitly described.

However, for certain specific applications such as to the solution of partial differential equations, it is often useful to investigate the optimality of spaces in Sobolev-type embeddings restricted to the context of Orlicz spaces. It turns out that the situation in this setting is significantly different than in the broader sense of r.i. spaces. For instance even the existence of an optimal Orlicz domain may fail.

In the talk we present an answer to the question in the special case when the target space is chosen from the class of the so-called Marcinkiewicz endpoint spaces. This is not as restrictive as it may seem since the most customary cases are covered. As a consequence of the method used in the approach we obtain similar results also for Sobolev trace embeddings of different orders and on various domains in $\mathbb{R}^n$ at once.

Generalized ordinary differential equations in metric spaces

Mgr. Břetislav Skovajsa

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Ročník: 1.
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Abstract

Recent years have revealed that many types of differential problems can be studied in the context of metric spaces i.e. without any prior linear structure. The theory of generalized ordinary differential equations has not been considered in this setting due to heavy reliance on integration over the target space. The versatility of the Henstock-Kurzweil integral allows this theory to cover a very wide range of problems; extending it to metric spaces puts all of them into a new perspective. This talk will be concerned with the basic principles behind the aforementioned generalization as well as the latest results and their expected impact.

Kernel methods in particle filter

Mgr. Ing. David Coufal, PhD.

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Ročník: 2.
Školitel: prof. RNDr. Viktor Beneš, DrSc.

Abstract

The particle filter is a tool that extends the use of the filtering techniques to non-linear and non-Gaussian processes. The work of the particle filter draws on sampling techniques in order to construct random measures that approximate conditional distributions of interest. These distributions are called the filtering distributions. The carriers of the random measures (in fact, the sampled points) are called particles. It is known that the constructed approximations converge to the filtering distributions with the increasing number of the particles generated.

In the talk, we aim on the construction of kernel estimates of the densities of the filtering distributions (the filtering densities). We will show that in spite of the non-i.i.d. character of the samples generated by the filter the kernel estimates still converge to the filtering densities. We will discuss the conditions and the character of the convergence along with the extension of the results to the convergence of derivatives of the estimates.

Cylindrical Volterra processes and stochastic evolution equations

Mgr. Petr Čoupek

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Ročník: 2.
Školitel: prof. RNDr. Bohdan Maslowski, DrSc.

Abstract

The aim of this talk is to present some results regarding a mild solution of a linear stochastic evolution equation of the form

$$\begin{aligned} \mathrm{d}X_t &= AX_t\,\mathrm{d}t + \phi\,\mathrm{d}B_t, \quad t \in [0,T], \\ X_0 &= x, \end{aligned} \tag{1}$$

which is driven by a cylindrical Volterra process – an infinite dimensional analogue to processes which admit the canonical representation $X_t = \int_0^t K(t,r)\mathrm{d}W_r$, where W denotes the standard Brownian motion and K is a suitable kernel. Our principal examples include the heat and wave equations and thus we introduce two sets of sufficient conditions under which the mild solution of (1) is a well-defined mean-square continuous Gaussian process. These findings extend our previous results and generalize the model with the driving process being a cylindrical fractional Brownian motion of Hurst parameter $H \geq 1/2$ (i.e. including the standard Brownian motion).

Ergodic control for infinite dimensional Lévy processes

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Ročník: 3.
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Abstract

In this contribution, weak solutions of stochastic evolution equations driven by square integrable infinite dimensional Lévy processes are studied. The commonly known Ito formula is directly applicable only to the strong solutions of the stochastic evolution equations. The assumption of the existence of the strong solution is however too restrictive, therefore the Ito formula in a suitable form for the weak solution of the stochastic evolution equation with the Lévy noise is given. The LQ stochastic optimal control problem as well as the corresponding ergodic control problem are formulated. Some ergodic control results in the case of square integrable Lévy noise are solved.

Gauss-Radau polynomial reconstruction of discontinuous Galerkin solutions

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Ročník: 1.
Školitel: prof. RNDr. Vít Dolejší, Ph.D., DSc.

Abstract

The space-time discontinuous Galerkin (STDG) method offers a robust and highly efficient approach to numerical solution of nonstationary PDE equations. We exploit the Gauss-Radau polynomial to design a reconstruction of the STDG solution, which is conforming with respect to time. This reconstruction can be conveniently used to show a relationship between the time discontinuous Galerkin method and a special class of Runge-Kutta methods. Due to this relation we can better understand the convergence behaviour of time discontinuous Galerkin method, especially the increased orders of convergence in the endpoints of the time intervals and also in the roots of Gauss-Radau polynomial. Furthermore, we propose a method how to employ this reconstruction to a posteriori estimation of the error of STDG method.

Interactive science centers

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Ročník: 3.
Školitel: prof. RNDr. Luboš Pick, CSc., DSc.

Abstract

The education of mathematics is usually associated with school teaching. However, the education is not just about the volume of knowledge, but mainly about a permanent mental mastery of skills and abilities to use them. An additional education of mathematics outside the classroom is thus developed in completely unconventional environments such as interactive science centers. Such centers have been appearing during the last few years around the world. In these centers, favorable conditions are created for the education of children. The children therefore can learn new skills while enjoying themselves in their spare time.

Tiling proofs of combinatorial identities involving Fibonacci, Lucas and Pell numbers

Mgr. Tereza Kovářová

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Ročník: 1.
Školitel: RNDr. Antonín Slavík, Ph.D.

Abstract

We present three combinatorial identities involving the well-known Fibonacci numbers, Lucas numbers and Pell numbers. These identities will be proved by means of a combinatorial argument. For that purpose we use the relationship between the Fibonacci numbers and the number of all possible tilings of a rectangle $1 \times n$ with 1×1 and 1×2 polyominoes, then the relationship between the Lucas numbers and the number of all possible tilings of a circular board composed of n cells with curved 1×1 and 1×2 polyominoes, and finally the relationship between the Pell numbers and the number of all possible tilings of a rectangle $1 \times n$ with blue 1×1, green 1×1 and 1×2 polyominoes.

Synthetic approach to Chasles theorem for timelike ruled surface

Mgr. Michal Zamboj

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Školitel: Mgr. Lukáš Krump, PhD.

Abstract

Chasles theorem provides a simple method for constructing tangent planes to a timelike ruled surface. Let Σ be a timelike ruled surface, and l be a line of Σ. Then points on l are in projectivity with tangent planes to Σ in these points. The theorem is highly applicable in descriptive geometry. We will present a synthetic approach to the proof of the theorem in the special case for the 2nd degree surfaces, as well as some further remarks on the synthetic approach to geometry in the projective extension of the Euclidean 3–dimensional space. For such purpose, we will introduce some graphic tools in software Wolfram Mathematica 10 and Geogebra 5.

Central limit theorem for Gibbsian U-statistics of facet processes

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Abstract

Special case of a Gibbsian facet process on a fixed window with a discrete orientation distribution and with increasing intensity of the underlying Poisson process is studied. All asymptotic moments for interaction U-statistics are calculated and using the method of moments the central limit theorem is derived.

Optimization of fuel cells

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Ročník: 1.
Školitel: prof. Ing. František Maršík, DrSc.

Abstract

An optimization of the fuel cells is a topic which importance grows with the fuel cells spread. Therefore, proper theoretical tools are necessary for correct interpretation of the experimental data and meaningful proposal of future development. The Exergy analysis is the most used theoretical tool for evaluation of the fuel cells efficiency, even though the fuel cells do not meet the Exergy analysis basic assumption, namely an isothermal boundary assumption. This inadequacy has been investigated in the article [1], where a new efficiency evaluation has been introduced. The new approach is illustrated on a simple solid oxide fuel cell model. The demonstration shows a substantial difference between the new approach and the Exergy analysis in the case of non-isothermal boundary.

[1] M. Pavelka, V. Klika, P. Vágner, and F. Maršík, Applied Energy 137, 158 (2015), ISSN 0306-2619, DOI: 10.1016/j.apenergy.2014.09.071