

FACULTY OF MATHEMATICS AND PHYSICS Charles University

26th Annual Student Conference Week of Doctoral Students

Book of Abstracts

of the

Week of Doctoral Students of the School of Mathematics 2017 June 15, 2017



Sokolovská 83 18675 Praha 8

Editace v LATEXu Mirko Rokyta, s použitím grafické úpravy Andreje Živčáka a Petra Pošty.

http://www.karlin.mff.cuni.cz/~rokyta/WDS-M/2017/ http://www.mff.cuni.cz/veda/konference/wds/

Vytisklo Reprostředisko MFF UK Praha © 2017 MFF UK Praha

Preface

In the beginning of 2014, the Management of the Faculty of Mathematics and Physics decided that the traditional conference of PhD students called the WDS (Week of Doctoral Students) would not be organized as an activity of the entire faculty. Instead, the decision as to whether to organize the conference or not was left to the respective Schools (of Computer Science, of Mathematics, and of Physics).

Already for the fourth year since then the School of Mathematics organizes its WDS-M (Week of Doctoral Students of the School of Mathematics, http://www.karlin.mff.cuni.cz/~rokyta/WDS-M/2017/), this time again as a one-day conference, in the framework, and as a continuation of, the (26th) WDS of the Faculty of Mathematics and Physics (http://www.mff.cuni.cz/veda/konference/wds/).

This year, mainly due to the activity of the *Charles University Chapter of SIAM* (http://siam.cuni.cz/), for which I am grateful, the program of the WDS-M has been extended by a poster session. Thanks to this, 24 students have registered as active participants to the conference. We believe that this event, which takes place in the "mathematical" Karlín building of the faculty, will attract the attention of the students but also of the broad mathematical audience. We thus encourage all of those interested in the scientific activities of our doctoral students to attend this meeting.

The conference is co-organized by the *School of Mathematics*, *Faculty of Mathematics and Physics*, *Charles University*, and *Charles University Chapter of SIAM*.

Prague, June 12, 2017

doc. RNDr. Mirko Rokyta, CSc. Vice-Dean for Mathematics Faculty of Mathematics and Physics Charles University Prague

Contents

Contributed talks

4M1 – Algebra, teorie čísel a matematická logika	
Mgr. Jan Grebík	4
4M2 – Geometrie a topologie, globální analýza a obecné struktury	
Mgr. Roland Púček	5
4M3 – Matematická analýza	
Mgr. Martin Michálek	6
RNDr. Vít Musil	7
Mgr. Jakub Slavík	8
4M6 – Vědecko-technické výpočty	
Mgr. Jakub Hrnčíř	9
4M8 – Obecné otázky matematiky a informatiky	
Mgr. Michal Zamboj	10
4M9 – Pravděpodobnost a statistika, ekonometrie a finanční matematika	
Mgr. Vít Kubelka	11
4F11 – Matematické a počítačové modelování	
Mgr. Michal Bathory	12
Mgr. Vojtěch Miloš	13
Mgr. Petr Pelech	14
Judith Stein	15

Posters

Mgr. Petr Zima (4M2)	16
Mgr. Václav Kryštof (4M3)	16
Mgr. Alena Skálová (4M3)	16
Mgr. Tomáš Gergelits (4M6)	17
Mgr. Marie Kubínová (4M6)	17
Mgr. Jan Kuřátko (4M6)	17
Mgr. Jakub Večeřa (4M9)	18
Mgr. Marek Čapek (4F11)	18
Mgr. Marek Netušil (4F11)	18
Mgr. Vít Orava (4F11)	19
Mgr. Helena Švihlová (4F11)	19
Mgr. Petr Vágner (4F11)	19

Borel ideals

Contributed talk

Mgr. Jan Grebík

E-mail: greboshrabos@seznam.cz *Obor studia:* 4M1 – Algebra, teorie čísel a matematická logika *Ročník:* 1. *Školitel:* RNDr. David Chodounský, Ph.D.

Abstract

We say that ideal on ω is Borel if it is a Borel subset of 2^{ω} . For example the ideal of finite subsets of ω , the sets of asymptotic density 0, the summable ideal $\{A \subseteq \omega : \sum_{n \in A} \frac{1}{n} < \infty\}$ or the ideal generated by cliques and anticliques in the Random graph. To study such objects we use the methods of infinite combinatorics and descriptive set theory. The aim of the talk is to present basic definitions and fundamental theorems as well as recent results and open questions. We define the Katětov order introduced by Katětov in 1968 to study convergence in topological spaces and mention Mazur's theorem that characterizes F_{σ} ideals in terms of lsc submeasures.

Problem of metrisability of parabolic geometries *Contributed talk*

Mgr. Roland Púček

E-mail: pucek.roland@gmail.com

Obor studia: 4M2 – Geometrie a topologie, globální analýza a obecné struktury *Ročník:* 1.

Školitel: prof. RNDr. Vladimír Souček, DrSc.

Abstract

In Riemannian geometry, the fundamental fact is that there exists a unique torsion-free connection (called the Levi-Civita connection) compatible with the Riemannian metric g, i.e. having the property $\nabla g = 0$. In projective geometry, the class of covariant derivatives defining the geometry is fixed and all these covariant derivatives have the same class of (non-parametrized) geodesics. Old (and non-trivial) problem is to find whether these curves are geodesics of a (pseudo-)Riemannian metric. Such projective structures are called metrizable. Surprisingly enough, U. Dini and R. Liuoville found in 19th century that the metrizability problem leads to a system of linear PDE's. In the last years, there were several papers dealing with these problems. The projective geometry is a representative example of the so called parabolic geometries (for full description, see the recent monograph by A. Čap and J. Slovák). It was realized recently that the corresponding linear metrizability operator is a special example of the so called first BGG operator.

In this more general context, the metrizability problem for (pseudo-) Riemannian geometries is naturally generalized to the sub-Riemannian situation. In the recent preprint, D. Calderbank, J. Slovák and V. Souček are discussing the classification of (real) *irreducible* parabolic geometries for which the linearisation method can be applied.

The aim of this talk is to formulate the linearisation method in a full generality and to provide a few examples.

Non-local transport equations and gradient flows in metric spaces Contributed talk

Mgr. Martin Michálek

E-mail: michalek@math.cas.cz *Obor studia:* 4M3 – Matematická analýza *Ročník:* 4. *Školitel:* prof. RNDr. Eduard Feireisl, DrSc.

Abstract

Some autonomous ordinary differential equations belong to the class of gradient flows:

$$\frac{\mathrm{d}}{\mathrm{d}t}\vec{x} = \nabla G(\vec{x}).$$

The result of Felix Otto (*Commun. Part. Diff. Eq.*, 1999) revealed that also some evolutionary partial differential equations admit a similar structure. To this end, one has to switch from the standard normed spaces of functions to subsets of non–negative measures supplemented by the Wasserstein metric. Along with the existence theory based on the geometry of sets of measures, we will present some applications.

Approximation of non-compact Sobolev embeddings

Contributed talk

RNDr. Vít Musil

E-mail: musil@karlin.mff.cuni.cz *Obor studia:* 4M3 – Matematická analýza *Ročník:* 3. *Školitel:* prof. RNDr. Luboš Pick, CSc., DSc.

Abstract

We will discuss various kinds of approximations of certain type of limiting Sobolev embedding into the space of continuous functions in terms of so-called *s*-numbers. All known results in this field always contain the compactness assumption. We will illustrate on an example that such at a first glance natural assumption is not justified in general and that it is reasonable to ask what happens on the non-compact borderline.

The crucial ingredient in our approach is the result from combinatorics and algebraic topology: the "zigzag" theorem. It says that every *k*-dimensional subspace of \mathbb{R}^n , $(k \le n)$, contains an element of the form (-1,1,-1,...) of length *k*. The proof in one-dimensional case also relies upon the linear ordering of the domain – an interval. In higher dimension we transfer this idea by using the approximation of space-filling curve, namely the Hilbert curve, for which we show that it "preserves locality" in some sense.



Let's play the snake!

Differential equations in unbounded domains

Contributed talk

Mgr. Jakub Slavík

E-mail: slavikj@karlin.mff.cuni.cz *Obor studia:* 4M3 – Matematická analýza *Ročník:* 4. *Školitel:* doc. RNDr. Dalibor Pražák, Ph.D.

Abstract

We study the asymptotic behaviour of evolution differential equations posed in the whole space \mathbb{R}^d and the complexity of the dynamics induced by the unboundedness of the spatial domain. We will discuss the proper choice of the space of initial data to include even nonintegrable functions and appropriate generalizations of fractal dimension and global and exponential attractors. Finally we will present a sufficient and necessary condition for the existence of an infinite dimensional exponential attractor for nonlinear reaction diffusion equation and discuss generalizations to other problems.

Operator preconditioning

Contributed talk

Mgr. Jakub Hrnčíř

E-mail: hrncir@karlin.mff.cuni.cz *Obor studia:* 4M6 – Vědecko-technické výpočty *Ročník:* 1. *Školitel:* prof. Ing. Zdeněk Strakoš, DrSc.

Abstract

The operator preconditioning is an attempt to provide an unified theoretical framework and background for preconditioning of iterative methods used in solution of systems of linear algebraic equations that arise from discretization of a boundary value problems for a partial differential equations (PDE). The preconditioning is often perceived as an ad hoc and heuristic transformation of the linear algebraic equations with the aim to speed up the convergence. The idea of the operator preconditioning is to find a proper preconditioner as a suitable operator on the infinite dimensional function spaces connected with the PDE operator formulation. The preconditioning on algebraic level is then obtained by discretization of the chosen operator. In the presentation I will introduce the context of solving a modelling problem, relevant important concepts and the some issues connected with the problem of preconditioning. Then the mathematical setting, basic tools, typical results, usage and scope of operator preconditioning will be presented.

Synthetic projective methods used for solving problems in geometry

Contributed talk

Mgr. Michal Zamboj

E-mail: zamboj@karlin.mff.cuni.cz *Obor studia:* 4M8 – Obecné otázky matematiky a informatiky *Ročník:* 3. *Školitel:* Mgr. Lukáš Krump, Ph.D

Abstract

In the contribution, we talk about synthetic methods in the projective extension of the real plane or three-dimensional space for solving problems of projective incidence and affine geometry. We use the concept of von Staudt's "Wurf", defined in his Beiträge zur Geometrie der Lage, and derived property that cross-ratios are invariant under projective transformations. The concept of choosing an infinite hyperplane is used for making hypothesis in an affine space to solve projective problems and vice-versa. Their mixtures with the analytic use of homogenous coordinates is applied on projective theorems. The insight into the von Staudt's constructions on the projective scale is given. The methods are shown on some examples in elementary planimetry and stereometry, proofs of Menelaus' and Ceva's theorems and applications of Pappus's theorem.

Stochastic differential equations and filtering theory

Contributed talk

Mgr. Vít Kubelka

E-mail: kubelka@karlin.mff.cuni.cz

Obor studia: 4M9 – Pravděpodobnost a statistika, ekonometrie a fin. matematika *Ročník:* 1.

Školitel: prof. RNDr. Bohdan Maslowski, DrSc.

Abstract

When treating linear differential equations driven by brownian motion, filtering problem can be solved by Kalman-Bucy filter. In finite dimension, Kalman-Bucy filter has been generalized for stochastic differential equations driven by fractional brownian motion. However, there is no contribution to this theory in Hilbert spaces.

First, a general idea of stochastic integration and stochastic differential equations will be mentioned. Then, linear filtering theory will be discussed and Kalman-Bucy filter for stochastic differential equations driven by fractional brownian motion in finite dimensional spaces will be presented. Finally, this problem will be formulated for Hilbert spaces and discussed.

Outflow boundary condition leading to minimal energy dissipation for an incompressible flow

Contributed talk

Mgr. Michal Bathory

E-mail: bathory@karlin.mff.cuni.cz *Obor studia:* 4F11 – Matematické a počítačové modelování *Ročník:* 1. *Školitel:* RNDr. Miroslav Bulíček, Ph.D.

Abstract

A method for determining the boundary condition on artificial boundaries is presented. This method is formulated as an optimization problem for appropriate functional representing the dissipation of energy. We show that this functional attains its minimum on the set of solutions to the Navier-Stokes system with unknown boundary condition on some part of the boundary. Thus, this method gives rise to a physically reasonable boundary condition which assures the existence of the corresponding solution. In particular, for the Stokes system, it is proved that the obtained implicit boundary condition implies the modification of the "do-nothing" boundary condition for the symmetric velocity gradient.

Thermodynamic analysis of the AC impedance spectroscopy of electrochemical cells

Contributed talk

Mgr. Vojtěch Miloš

E-mail: vojta.milos@gmail.com *Obor studia:* 4F11 – Matematické a počítačové modelování *Ročník:* 1. *Školitel:* prof. Ing. František Maršík, DrSc.

Abstract

Electrochemical impedance spectropy is an important experimental technique used for investigating properties of electrochemical cells, particulary fuel cells or electrolysers. The method is based on applying a sinusoidal voltage signal on a cell and comparing it with the current response (or vice versa). This is done for a wide range of frequencies. Results are often displayed by Nyquist plot. Although impedance spectroscopy can reveal partial processes in the cell, there is a great space for research - how to interpret obtained data. A standard way is to construct an auxilary circuit consisting of eletrotechnical components (resistor, capacitor etc.), but this method does not give a sufficient insight to processes taking place in the cell. However, there are already results close to this topic in terms of nonequilibrium termodynamics such as a doctoral thesis of Michal Pavelka, analytical models of A. A. Kulikovski and also a research in Weierstraß-Institut in Germany. In our contribution, we provide an overview of existing termodynamic models and sketch out options for further research.

Some remarks and ideas about stress tensors in peridynamics and their use in comparison with classical elasticity

Contributed talk

Mgr. Petr Pelech

E-mail: petr.pelech@gmail.com *Obor studia:* 4F11 – Matematické a počítačové modelování *Ročník:* 1. *Školitel:* doc. RNDr. Martin Kružík, Ph.D.

Abstract

Peridynamics is a non-local model in continuum mechanics introduced by Silling. The non-locality is reflected in the fact that points at a finite distance exert force upon each other. If, however, these points are more distant than a characteristic length called horizon, it is customary to assume that they do not interact. Hence disjoint parts of the continuum may interact through nearby volumes. This is in contrast to local continuum models, where two adjacent parts of the deformed body interacts through a common surface and the stress is a fundamental concept. In order to provide a connection to local theories, some notions of stress in peridynamics has been defined. In my presentation I will suggest another definition of a peridynamic stress tensor, which unifies the earlier approaches and hopefully clarifies some of their ambiguities.

Stretching of the vitreous body

Contributed talk

Judith Stein

E-mail: judith.stein@iwr.uni-heidelberg.de *Obor studia:* 4F11 – Matematické a počítačové modelování *Ročník:* 2. *Školitel:* Mgr. Vít Průša, Ph.D.

Abstract

Although linked to several vitreoretinal pathologies the material behavior of the vitreous body in response to mechanical loads is not well understood. In this study, we analyze the deformation of the eye, especially the vitreous body which shows a viscoelastic behavior due to a network of collagen fibers. According to the literature this behavior can be described by the viscoelastic Burgers model for which we show two equivalent descriptions and compare the parameters. Finally, we reproduce an experiment from literature by using Finite Element simulations. Therefore, we add the lens and sclera as compressible elastic neo-Hookean solids to our vitreous model. In order to compare a healthy versus a pathological vitreous with liquefaction or complete vitrectomy, we compare the viscoelastic versus the viscous Navier-Stokes model. Our simulations show that due to the very elastic sclera the force and flow fields are nearly the same for both models, but there is a significant difference in stresses.

Killing equations and similar overdetermined systems of PDEs Poster Mgr. Petr Zima

E-mail: zima@karlin.mff.cuni.cz *Obor studia:* 4M2 – Geometrie a topologie, globální analýza a obecné struktury *Ročník:* 3. *Školitel:* doc. RNDr. Petr Somberg, Ph.D.

Ilmanen lemma

Poster

Mgr. Václav Kryštof

E-mail: krystof@karlin.mff.cuni.cz *Obor studia:* 4M3 – Matematická analýza *Ročník:* 1. *Školitel:* prof. RNDr. Luděk Zajíček, DrSc.

Gradient mapping of functions of several variables

Poster

Mgr. Alena Skálová

E-mail: skalova.aja@gmail.com *Obor studia:* 4M3 – Matematická analýza *Ročník:* 3. *Školitel:* doc. RNDr. Miroslav Zelený, Ph.D.

Composite polynomial convergence bounds in finite precision CG computations

Poster Mgr. Tomáš Gergelits

E-mail: gergelits@karlin.mff.cuni.cz *Obor studia:* 4M6 – Vědecko-technické výpočty *Ročník:* 4. *Školitel:* prof. Ing. Zdeněk Strakoš, DrSc.

Rank deficiency of Krylov subspaces in FP computations Poster

Mgr. Marie Kubínová

E-mail: kubinova@karlin.mff.cuni.cz *Obor studia:* 4M6 – Vědecko-technické výpočty *Ročník:* 4. *Školitel:* RNDr. Iveta Hnětynková, Ph.D.

The use of sequential quadratic programming for solving reachability problems

Poster

Mgr. Jan Kuřátko

E-mail: kuratko@karlin.mff.cuni.cz *Obor studia:* 4M6 – Vědecko-technické výpočty *Ročník:* 5. *Školitel:* Ing. Stefan Ratschan, Ph.D.

Estimation of planar segment process Poster Mgr. Jakub Večeřa

E-mail: vecera@karlin.mff.cuni.cz *Obor studia:* 4M9 – Pravděpodobnost a statistika, ekonometrie a fin. matematika *Ročník:* 3. *Školitel:* prof. RNDr. Viktor Beneš, DrSc.

Blood flow modelling

Poster

Mgr. Marek Čapek

E-mail: capem2am@karlin.mff.cuni.cz *Obor studia:* 4F11 – Matematické a počítačové modelování *Ročník:* 7. *Školitel:* RNDr. Ing. Jaroslav Hron, PhD.

Multiscale modelling of aortic media

Poster

Mgr. Marek Netušil

E-mail: marek.netusil@gmail.com *Obor studia:* 4F11 – Matematické a počítačové modelování *Ročník:* 5. *Školitel:* prof. Ing. František Maršík, DrSc.

Multi-phase modelling of reactive flow in fluidized bed reactors *Poster* Mgr. Vít Orava

E-mail: orava@karlin.mff.cuni.cz *Obor studia:* 4F11 – Matematické a počítačové modelování *Ročník:* 4. *Školitel:* RNDr. Ing. Jaroslav Hron, Ph.D.

Computation of the pressure drop and the disipated energy in narrowed pipes with a view towards its application in cardiovascular mechanics

Poster

Mgr. Helena Švihlová

E-mail: svihlova@karlin.mff.cuni.cz *Obor studia:* 4F11 – Matematické a počítačové modelování *Ročník:* 4. *Školitel:* RNDr. Ing. Jaroslav Hron, Ph.D.

Dusty gas model in the framework of Extended Irreversible Thermodynamics

Poster

Mgr. Petr Vágner

E-mail: affro@atrey.karlin.mff.cuni.cz *Obor studia:* 4F11 – Matematické a počítačové modelování *Ročník:* 3. *Školitel:* prof. Ing. František Maršík, DrSc.

Remarks and notes