N_{ij} = observed count $\qquad (\pi_{ij})\ (i,j) \equiv (X=i,\ Z=j)$

---

sampling mechanism

Poisson

(X, Z) = explanatory variables

response = N_{ij} = count if $(X=i,\ Z=j)$

$\mathbb{E} N_{ij} = m_{ij}$ $\qquad , n = \sum_i \sum_j N_{ij}$ (random)

Multinomial (without fixed margins)

response = (X, Z) $\qquad \pi_{ij} = P(X=i,\ Z=j)$

$N_{ij} = \#(X=i, Z=j)$ in a sample of size n

$\mathbb{E} N_{ij} = \underbrace{n \cdot \pi_{ij}}_{=:\, m_{ij}} \quad (= \underbrace{(\mathbb{E} n)}_{m_{++}} \pi_{ij})$

---

log-linear model :

$$\log m_{ij} = \eta_{ij} \text{ (linear predictor)}$$

$$\log \pi_{ij} = \underbrace{- \log m_{++}}_{-\log n} + \eta_{ij}$$

$$\pi_{ij} = \frac{e^{\eta_{ij}}}{m_{++}} \quad \Longrightarrow$$

$\Rightarrow$

$$\frac{\pi_{A,B}}{\pi_{C,D}} = \exp\left(\eta_{A,B} - \eta_{C,D}\right)$$

!!! interpretation of params. of log-lin. model

$\equiv$ interpretation of linear model (ANOVA-like)

IMPORTANT 1

$$\frac{P(X=i_1, Z=j)}{P(X=i_2, Z=j)} = \frac{P(X=i_1 \mid Z=j)}{P(X=i_2 \mid Z=j)} =: odds_X(i_1, i_2 \mid Z=j)$$

$$= \exp(\underbrace{\eta_{i_1,j} - \eta_{i_2,j}}_{})$$

just linear combination of β's

(or perhaps one of β's)

IMPORTANT 2

$$\frac{\dfrac{P(X=i_1, Z=j_1)}{P(X=i_2, Z=j_1)}}{\dfrac{P(X=i_1, Z=j_2)}{P(X=i_2, Z=j_2)}} = \frac{\dfrac{P(X=i_1 / Z=j_1)}{P(X=i_2 / Z=j_1)}}{\dfrac{P(X=i_1 / Z=j_2)}{P(X=i_2 / Z=j_2)}} =$$

$$= \frac{odds_X(i_1, i_2 / Z=j_1)}{odds_X(i_1, i_2 / Z=j_2)} = OR_X(i_1, i_2 / Z: j_1 \leftarrow j_2)$$

$$= \frac{\exp(\eta_{i_1, j_1} - \eta_{i_2, j_1})}{\exp(\eta_{i_1, j_2} - \eta_{i_2, j_2})} = \exp(\underbrace{\ldots\ldots}_{\text{just linear combination of } \beta\text{'s}})$$

Saturated model: ($\equiv$ ANOVA two-way with interactions)

$$\log m_{ij} = \beta_0 + \beta_i^X + \beta_j^Z + \beta_{ij}^{XZ}$$

identif. constraints

e.g. $\beta_1^X = 0$, $\beta_1^Z = 0$, $\beta_{1,j}^{XZ} = 0 \quad \forall j$

$\beta_{i,1}^{X,Z} = 0 \quad \forall i$

$\equiv$ X and Z parameterized through reference group pseudocontrasts

Meaning of β's?

$\rightarrow$ go back to Linear model

just $\exp(\dots) \longrightarrow$ odds,

or Odds ratio