# Bayes Factor

Bayes model: $p(y|\theta)$, $p(\theta)$

$$p(y,\theta) = p(y|\theta)\cdot p(\theta)$$

BŮH $\theta$ $\to$ $y$

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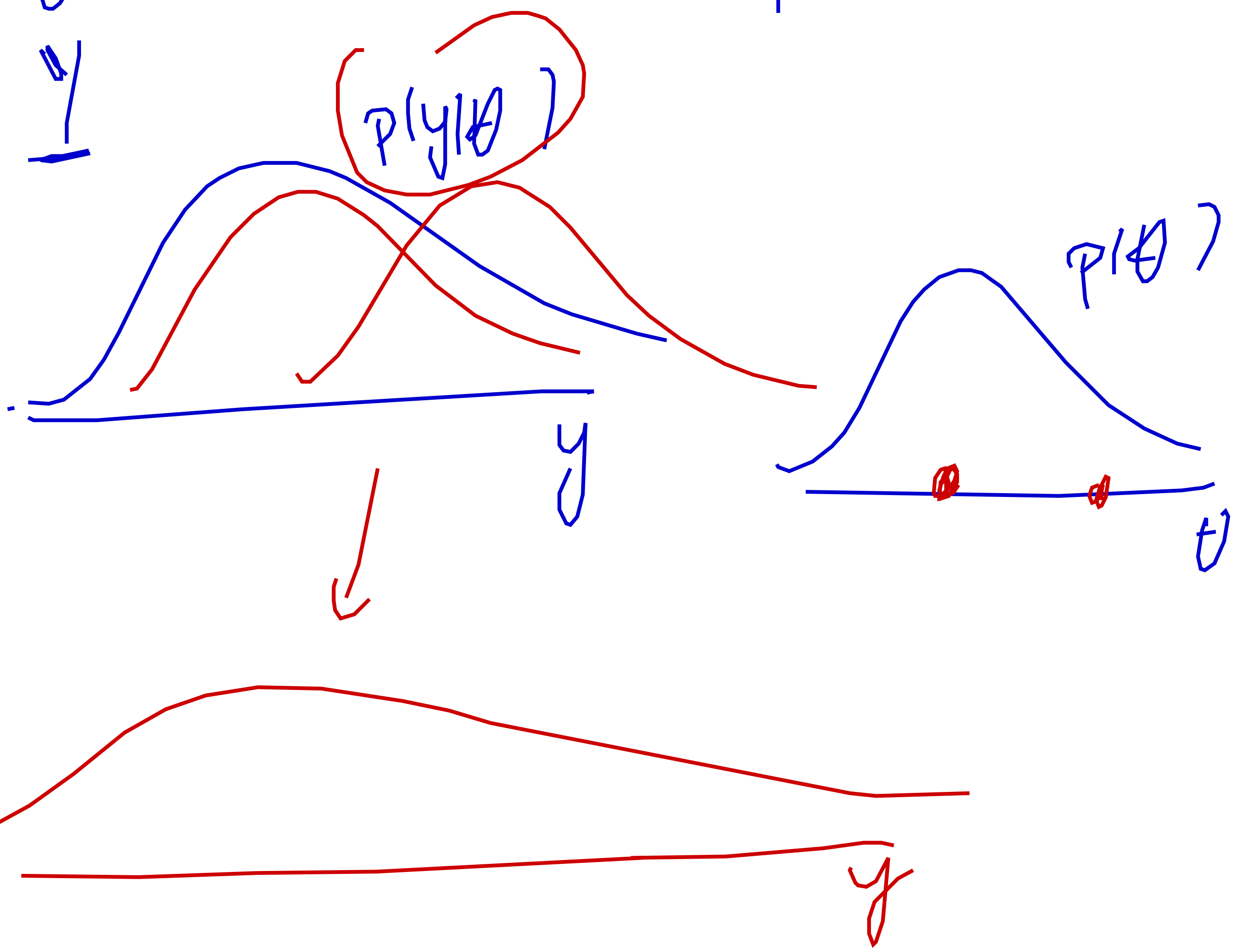


uvažované modely $M_1, \dots, M_k$

$p_k(y|\theta_k)$, $p_k(\theta_k)$ $\longrightarrow$ $p_k(y) = p(y|M_k)$

každý model → mám představu, jak je možný

$P(M_1), \ldots, P(M_r)$: aprior prsti. jednotl. modelů

→ Bayes. věta

$$P(M_k \mid y) \propto p(y \mid M_k) \cdot P(M_k)$$

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Bayes factor

$$BF(M_k, M_l) = \frac{p(y \mid M_k)}{p(y \mid M_l)}$$

$$p_k(y) = \int p(y \mid \theta_k)\, p(\theta_k)\, d\theta_k =$$

$$= \mathbb{E}_{p(\theta_k)}\, p(y \mid \theta_k)$$

co dělat, pokud $p(\theta_k) \propto 1$

$\mathbb{E}_m$ nemusí existovat

Weakly informative, např.

$\tau$ = inverz. rozptyl $p(\tau) = Ga(\varepsilon, \varepsilon)$

$\varepsilon \to 0$

## Pro aposteriorní prediktivní rozdělení

model: $p(y|\theta)$, $p(\theta)$

$$\rightarrow \underline{p(y)} = \int p(y|\theta)\,p(\theta)\,d\theta$$

$y_{new} \sim p(y)$

předp. $y \perp y_{new}$ za podm. $\theta$

$$p(y_{new}|y) = \int p(y_{new}, \theta | y)\, d\theta =$$

$$= \int p(y_{new} | \cancel{y}, \theta)\, p(\theta|y)\, d\theta =$$

$$= \int \underbrace{p(y_{new}|\theta)}_{\text{věrohodn. modelu}}\, p(\theta|y)\, d\theta$$

SROVNEJ:

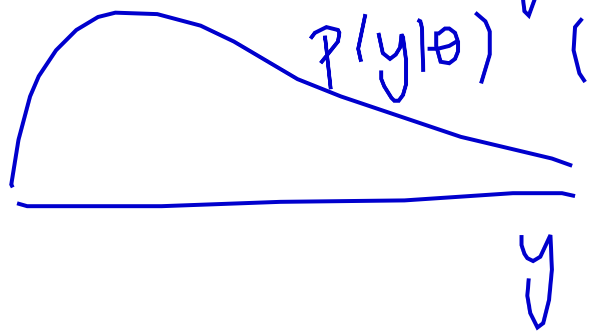
$$p(y) = \int p(y|\theta)\, \underline{p(\theta)}\, d\theta = E_{p(\theta)}\, p(y|\theta)$$

$$p(y_{new}|y) = \int p(y_{new}|\theta)\, \underline{p(\theta|y)}\, d\theta$$

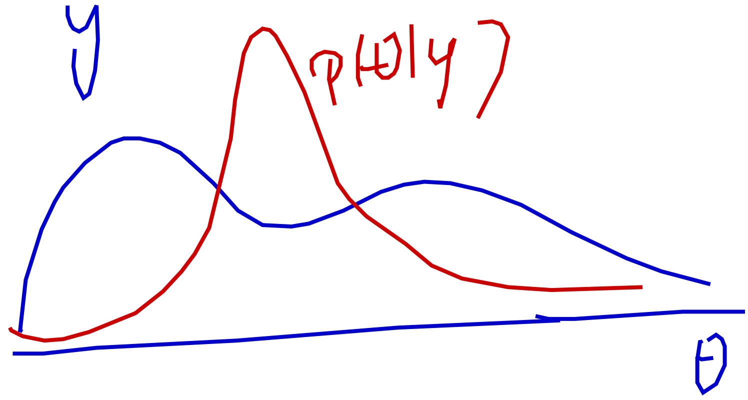
$$= E_{p(\theta|y)}\, p(y_{new}|\theta)$$

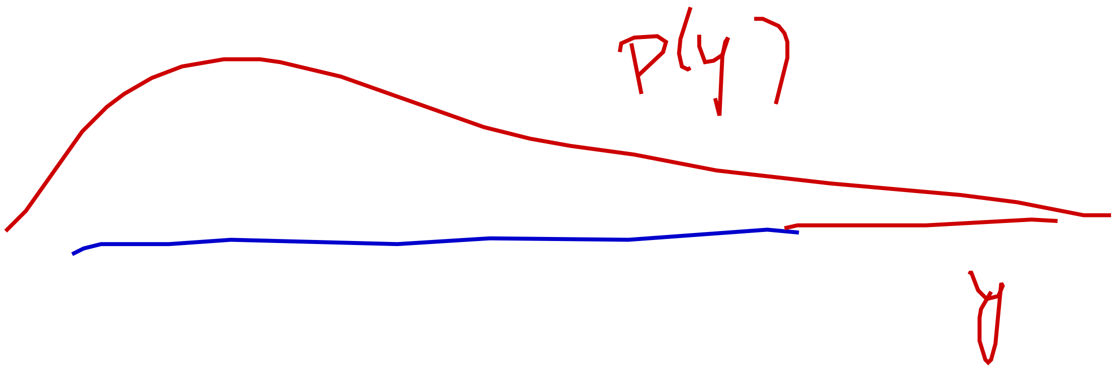
vážený průměr věrohodností,
kde váhy = aposter. rozděl.

$p(y|\theta)$ (MODEL, věrohodnost)

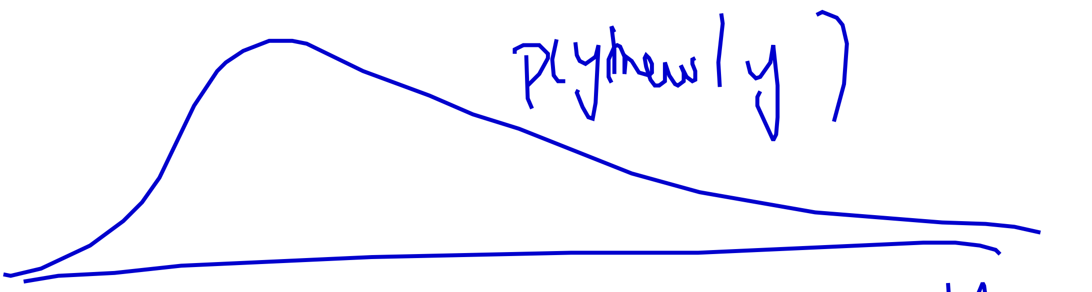



apriori:




DATA → $p(\theta|y)$




vhodné k predikování

$$KL(Q_2, Q_1) = E_{Q_1} \log \frac{q_1(y)}{q_2(y)}$$

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$Q_1 \equiv Q \equiv$ skutečné rozdíl. dat

$\equiv P(y)$

$Q_2 \equiv p(y|\theta)$ (věrohodnost klasick. modelu)