• time transformation: $Z(t) = g(t)$ where g is some known/chosen function

– no interaction model: $\lambda(t|\underline{Z}) = \lambda_0(t)\cdot\exp\{\alpha g(t) + \beta^T Z\}$

Partial Likelihood:
$$\prod_{i=1}^{n}\prod_{s>0}\left[\frac{Y_i(s)\cdot\exp\{\alpha g(s)+\beta^T Z_i\}}{\sum_{j=1}^{n} Y_j(s)\cdot\exp\{\alpha g(s)+\beta^T Z_j\}}\right]^{\Delta N_i(s)} = \prod_{i=1}^{n}\prod_{s>0}\left[\frac{Y_i(s)\exp\{\beta^T Z_i\}}{\sum_{j=1}^{n} Y_j(s)\exp\{\beta^T Z_j\}}\right]^{\Delta N_i(s)}$$

⇝ does not affect the model at all!

– must be in interaction: $\lambda(t|\underline{Z}) = \lambda_0(t)\cdot\exp\{\alpha g(t) + \beta\cdot Z + \gamma\, g(t) Z + \omega W\}$ to make some difference

PL:
$$\prod_{i=1}^{n}\prod_{s>0}\left[\frac{Y_i(s)\exp\{\alpha g(s)+\beta Z_i+\gamma g(s) Z_i+\omega W_i\}}{\sum_{j=1}^{n} Y_j(s)\exp\{\alpha g(s)+\beta Z_j+\gamma g(s) Z_j+\omega W_j\}}\right]^{\Delta N_i(s)} = \prod_{i=1}^{n}\prod_{s>0}\left[\frac{Y_i(s)\exp\{\beta Z_i+\gamma g(s) Z_i+\omega W_i\}}{\sum_{j=1}^{n} Y_j(s)\exp\{\beta Z_j+\gamma g(s) Z_j+\omega W_j\}}\right]^{\Delta N_i(s)}$$

⇝ α is meaningless in this parametrization → only reparametrizes the baseline hazard, but cannot be estimated

baseline hazard: $Z = 0 = W$ ⇝ $\lambda(t|\underline{Z}) = \lambda_0(t)\cdot\exp\{\alpha g(t)\}$

$\boxed{\beta+\gamma}$: $\dfrac{\lambda(t|Z+1, W)}{\lambda(t|Z, W)} = \dfrac{\exp\{\beta(Z+1)+\gamma g(t)(Z+1)+\omega W\}}{\exp\{\beta Z+\gamma g(t) Z+\omega W\}} = e^{\beta+\gamma g(t)}$ ← effect of unit increase in Z
"time-varying coefficient" $\beta(t)$

$\boxed{\omega}$: $\dfrac{\lambda(t|Z, W+1)}{\lambda(t|Z, W)} = \exp\{\omega\}$ ← multiplicative effect of unit increase in W

in Ⓡ: `coxph(Surv(time, delta) ~ Z + tt(Z) + W, data, tt = function(x, t, ...){x · g(t)})`

Warning: be careful about the units of time if g is meant to be for different units