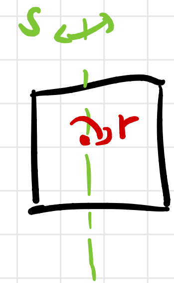
10 Mojitos?

$$\underline{D}_8 = \langle r, s \mid r^4 = s^2 = 1,\ s^{-1} r s = r^{-1} \rangle$$



$$C(D_8) = [D_8, D_8] = \{1, r^2\}$$

$\underline{D}_8 = (D_8, \cdot)$ satisfies the identities

$$[x,y] = x^{-1}y^{-1}xy$$

$$x^4 \approx 1, \quad [x,y]^2 \approx 1, \quad z \cdot [x,y] \approx [x,y] z$$

$\Rightarrow$ every term $t \in Clo(\underline{D}_8)$ has normal form

$$t(x, 1, \ldots, 1) = x^{\alpha_1}$$

$$t(x_1, x_2 \ldots x_n) \approx x_1^{\alpha_1} \cdot \ldots \cdot x_n^{\alpha_n} [x_1, x_2]^{\beta_{12}} \cdot [x_1, x_3]^{\beta_{13}} \cdot \ldots \cdot [x_{n-1}, x_n]^{\beta_{n-1,n}}$$

$$\alpha_i \in [4] \qquad \beta_{ij} \in [2]$$

$$t(x_1, x_2 \dots x_n) \approx x_1^{\alpha_1} \cdots x_n^{\alpha_n} [x_1, x_2]^{\beta_{12}} \cdot [x_1, x_3]^{\beta_{13}} \cdots [x_{n-1}, x_n]^{\beta_{n-1,n}}$$

$$\alpha_i \in [4] \qquad \beta_{ij} \in [2]$$

$[x,y]^2 = 1$

$[y,x] = [x,y]^{-1}$

Claim: $\nexists\, t \in Clo(\underline{D}_8)$, idp., block-symm. $t(\overleftrightarrow{x_1 \dots x_\ell}, \overleftrightarrow{x_{\ell+1} \dots x_{2\ell+1}})$

Pf: 1) $x^{\alpha_1} = t(\overleftrightarrow{x\, 1 \dots 1}, 1 \dots 1) \approx x^{\alpha_i} \Rightarrow \begin{cases} \alpha_i = \alpha & \forall i \leq \ell \\ \alpha_i = \gamma & \forall i > \ell \end{cases}$

2) $x = t(x \dots x) = x^{\ell \cdot \alpha + (\ell+1)\gamma} \Rightarrow \alpha \in \{\pm 1\}$ or $\gamma \in \{\pm 1$

3) wlog $\alpha = 1$ $\quad t(x\, y\, 1 \dots 1, 1 \dots 1) = t(y\, x\, 1 \dots 1, 1 \dots 1)$

$$\Rightarrow x y [x,y]^{\beta_{12}} = y x [y,x]^{\beta_{12}} \Rightarrow xy = yx \text{ ↯}$$

[AIMM'14] $\Rightarrow \exists\, \mathbb{D} = (D_8, R): \mathrm{Pol}(\mathbb{D}) = Clo(D_8, xy^{-1}z)$ not solvable by BLP+AIP

$$t(x_1, x_2 \dots x_n) \approx x_1^{\alpha_1} \cdot \dots \cdot x_n^{\alpha_n} [x_1, x_2]^{\beta_{12}} \cdot [x_1, x_3]^{\beta_{13}} \cdot \dots \cdot [x_{n-1}, x_n]^{\beta_{n-1,n}}$$

$$\alpha_i \in [4] \qquad \beta_{ij} \in [2]$$

Claim $\nexists\, t \in Clo(D_8)$ idp. weak $(11, 11 \dots 11)$-symmetric

$$t(\underbrace{x_1 \dots x_{11}}_{\alpha_1}, \underbrace{x_{12} \dots x_{22}}_{\alpha_{12}}, \dots \underbrace{x_{121}}_{\alpha_{111}})$$

I) α_i only depend on blocks

$$„t(x\, 1 \dots 1, x \dots x, 1 \dots 1) = x^{\alpha_1} \cdot \cancel{x^c}$$
$$t(1\, x\, 1 \dots 1, x \dots x, 1 \dots 1) = x^{\alpha_2} \cdot \cancel{x^c}$$
$$\Rightarrow \alpha_1 = \alpha_2$$

II) $x \approx t(x \dots x) \Rightarrow \exists i : \alpha_i \in \{\pm 1\}$

wlog. $\alpha_1 = 1$

III) $\underline{t}(x, y, 1 \dots 1, \overbrace{x \dots x}^{A}, \overbrace{y \dots y}^{B}, 1 \dots 1) =$

$$x y \cdot x^{C} \cdot y^{D} \cdot \prod_{i \in B} [x, y]^{\beta_{1,i}} \cdot \prod_{j \in A} [x, y]^{\beta_{2j}} \prod_{\substack{k \in A \\ m \in B}} [x, y]^{\beta_{k,m}}$$

$\beta_{i,M} := \sum_{j \in M} \beta_{i,j}$

$$= x y \cdot x^{C} y^{D} \cdot [x, y]^{\beta_{1B} + \beta_{2A} + \beta_{AB}}$$

$$= \underline{t}(y x, 1 \dots 1, x \dots x, y \dots y, 1 \dots 1) =$$

$$y x \cdot x^{C} \cdot y^{D} \cdot [x, y]^{\beta_{1A} + \beta_{2B} + \beta_{AB}}$$

$$\Rightarrow \quad x y [x, y]^{\beta_{1B} + \beta_{2A}} = y x [x, y]^{\beta_{1A} + \beta_{2B}}$$

$$\Rightarrow \quad \beta_{1A} + \beta_{2B} + \beta_{1B} + \beta_{2A} = 1 \quad \text{in } \mathbb{Z}_2$$

Similarly for

$$t(1\,x\,y\,1 \ldots 1,\ *) \simeq t(1\,y\,x\,1 \ldots 1,\ *)$$

$$t(x\,1\,y\,1 \ldots 1,\ *) \simeq t(y\,1\,x\,1 \ldots 1,\ *)$$

$$\Rightarrow \begin{cases} \beta_{1A} + \beta_{2A} + \beta_{1B} + \beta_{2B} = 1 \\ \beta_{2A} + \beta_{3A} + \beta_{2B} + \beta_{3B} = 1 \\ \beta_{1A} + \beta_{3A} + \beta_{1B} + \beta_{3B} = 1 \end{cases}$$

$$0 = 1 \quad ↯$$

$\underline{D}_8 = (D_8, xy^{-1}z)$

only Mal'tsev terms

$xy^{-1}z \quad zy^{-1}x$

$\Rightarrow \underline{D}_8$ minimal Mal'tsev

If $\mathcal{C} \subseteq Clo(\underline{D}_8)$

Mal'tsev $\quad \mathcal{C} = Clo(\underline{D}_8)$

solvable $\Rightarrow$ min. Taylor

$10\frac{1}{2}$ Mojitos?

Question (Benedikt): Is a group-coset CSP not solvably by k-cons. + AIP if G abelian? (∧ not)

Let G be 2-nilpotent, odd order.

$m := \exp([G,G])$ „Baer trick"

then $x+y := x \cdot y \cdot [x,y]^{\frac{m-1}{2}}$ is abelian group multiplication

E.g.

$$y+x = y \cdot x \cdot [y,x]^{\frac{m-1}{2}} = x \cdot y \cdot [y,x]^{\frac{m+1}{2}}$$

$$= x \cdot y\, [x,y]^{-\left(\frac{m+1}{2}\right)} = x \cdot y\, [x,y]^{\frac{m-1}{2}} = x+y$$

Then $x - y + z = x y^{-1} z [x,z]^{m\frac{-1}{2}} [x,y]^{m\frac{+1}{2}} [y,z]^{m\frac{+1}{2}}$

$\in Clo(G, \cdot)$, idempotent

$x_1 - x_2 + x_3 - x_4 + \dots + x_{2n+1} \in Clo(G, \cdot)$

is alternating term

$\Rightarrow$ $\exists \mathbb{A}$: $Pol(\mathbb{A}) = Clo(G, \cdot)^{id}$,

CSP($\mathbb{A}$) is solvable by AIP.

in fact we can get $x - y + z \in Clo(x y^{-1} z, y^{-1} x y)$

$\Rightarrow$ $\mathbb{A} = (A, R_1 \dots R_m)$ where $R_i = a H$, $H \trianglelefteq G^{m_i}$