Universal Algebra 2 - Exercises 5

April 25, 2019

1. Let $H = (V; E)$ be an (undirected) bipartite graph, i.e. a graph such that $V = V_1 \cup V_2$ such that the only edges are between vertices from V_1 and V_2.

    - Prove that CSP(H) is equivalent to the two-coloring problem CSP(K_2) (which is in P)
    - Show that K_2 has a majority polymorphism, i.e. a ternary $m(x, y, z)$ satisfying $m(x, x, y) \approx m(x, y, x) \approx m(y, x, x) \approx x$.

2. Prove that the algebra $\mathbf{A} = (A, \cdot)$ with the operation table table does not have a Taylor term.

$\cdot$	0	1	2
0	0	0	2
1	0	1	2
2	0	2	2

3. Let $R \subseteq A \times A$ be subdirect (i.e. its projections to the first / second coordinate are equal to A). Show that if R is invariant under some semilattice operation on A, then every pp-sentence $\exists x_1, \ldots, x_n R(x_{i_1}, x_{j_1}) \wedge \cdots \wedge R(x_{i_k}, x_{j_k})$ is true in A.
(*) Find a counterexample to the statement, assuming that R is invariant under some other Taylor operation.

4. Let $\mathcal{C}$ be the clone generated by all injective operations from ω^n to ω for all $n \geq 1$. Show that $\mathcal{C}$ does not contain any Taylor operation. Prove that, on the other hand $\mathcal{C}$ has no clone homomorphism to the projections by finding some other non-trivial identity holding in $\mathcal{C}$ (Hint: bijections $\omega^2 \to \omega$).