

Universal Algebra 2 - Homework 2

due on ~~April 18th 2019~~ April 25th 2019

1. (10 points) Find a convergent rewriting system equivalent to the identity $y \approx (y \cdot x) \cdot (z \cdot y)$ (Hint: the Knuth-Bendix algorithm outputs a system with 3 identities)
2. ~~(10 points~~ (*) Let us consider the axioms for idempotent semigroups: $\mathcal{E} = \{x \cdot (y \cdot z) \approx (x \cdot y) \cdot z, x \cdot x \approx x\}$. ~~Find a convergent rewriting system equivalent to $\mathcal{E}$.~~ Show that $\mathcal{E}$ is finitely terminating. Is there an equivalent rewriting system that is convergent?
3. (10 points) Show that for every $n > 1$ the cyclic group $\mathbb{Z}_n$ is finitely based, by giving an explicit equational basis. Show that the size of subdirect irreducible elements of $HSP(\mathbb{Z}_n)$ is bounded by a constant $n \in \mathbb{N}$ (you may also use facts from Universal Algebra 1 to answer this question).