NMAI076 - Algebra 2 - spring semester 2023/24

Homework 1

Deadline 21.3.2024, 15:40

Exercise 1. (10 points) Let $G = \{f\colon \mathbb{R} \to \mathbb{R}\colon f(x) = ax + b,\ a, b \in \mathbb{R}, a \neq 0\}$.

(1.1) Prove that G is a group with respect to the composition of functions.

(1.2) Prove that $N = \{f\colon \mathbb{R} \to \mathbb{R}\colon f(x) = x + b,\ b \in \mathbb{R}\}$ is a normal subgroup of G.

(1.3) Describe the quotient G/N.

Exercise 2. (10 points) Consider the dihedral group D_8 (the group of symmetries of the square, or equivalently $D_8 = \langle (1234), (14)(23) \rangle \leq S_4$).

(2.1) Determine the order of D_8 and the order of each of its elements.

(2.2) Determine (up to isomorphism) all homomorphic images of D_8.

Exercise 3. (10 points) Prove that every group G of order 6 is isomorphic to $\mathbb{Z}_6$ or S_3.

Hint: Note that G must be isomorphic to $\mathbb{Z}_6$ if it contains an element of order 6, or if G is abelian (Why?). If this is not the case, show that $G = \langle a, b \rangle$ with $ord(a) = 3$, $ord(b) = 2$ and $ab \neq ba$. Use this to show that $G \cong S_3$.