Convex Optimization 2025/26

Practical session # 3

October 16, 2025

1. Show that $f(x) = \log\log(x)$ is concave on $\mathrm{dom}(f) = (1, \infty)$. What about $f(x) = \log\log\ldots\log(x)$?

2. Find two convex functions $g, h\colon \mathbf{R} \to \mathbf{R}$, such that $g \circ h$ is not convex.
   Can you find an example where h is non-decreasing?

3. Is every convex function $f\colon \mathbf{R}^n \to \mathbf{R}$ continuous on its domain? (*) What if $\mathrm{dom}(f)$ is open?

4. Consider the linear program:

$$\begin{aligned} \text{minimize } & f(x_1, x_2, x_3) = 2x_1 + 3x_2 - x_3 \\ \text{subject to } & x_1 - x_2 \geq 4 \\ & 3x_1 + x_2 \leq 1 \\ & x_3 \leq 0 \end{aligned}$$

   Find an equivalent linear program of the form:

$$\begin{aligned} \text{minimize } & c^T y \\ \text{subject to } & Ay = b \\ & y \succeq 0 \end{aligned}$$

   (Hint: start by turning existing inequality constraints into an equality constraint with the help of an additional non-negative variable.)

5. Let $f\colon \mathbf{R}^{n+m} \to \mathbf{R}$ be a convex function, and let $C \subseteq \mathbf{R}^m$ be a convex set. Show that $g(x) = \inf_{y \in C} f(x, y)$ is also convex. (Hint: describe $\mathrm{epi}(g)$. Observe that projection of convex sets are convex sets.)

   As a consequence the distance function $x \mapsto \mathrm{dist}(x, C) = \inf_{y \in C} \|x - y\|$ is a convex function for every convex set C.