Convex Optimization 2025/26

Practical session # 8

November 20, 2025

1. Consider the QCQP

$$\begin{aligned} &\text{minimize } x_1^2 + x_2^2 \\ &\text{subject to } (x_1-1)^2 + (x_2-1)^2 \leq 1 \\ &\qquad\qquad (x_1-1)^2 + (x_2+1)^2 \leq 1 \end{aligned}$$

   - Sketch the feasible set and find optimal point x^* and optimal value p^*.
   - Give the KKT conditions. Are there Langrange multipliers λ_1^* and λ_2^* that prove that x^* is optimal?
   - Does strong duality hold?

2. Let us consider an LP in normal form:

$$\begin{aligned} &\text{minimize } c^T x \\ &\text{subject to } Ax = b \\ &\qquad\qquad x \succeq 0 \end{aligned}$$

   Show that it is equivalent to the dual of its dual.

3. Consider the feasibility problem for the above LP, so

$$\begin{aligned} &\text{minimize } 0 \\ &\text{subject to } Ax = b \\ &\qquad\qquad x \succeq 0 \end{aligned}$$

   Derive Farkas' lemma from using Exercise 2 and strong duality for LPs.

4. For each of the following problems draw a sketch of $\mathcal{A} = \{(u,t) \mid \exists x \in \mathcal{D} : f_0(x) \leq t, f_1(x) \leq u\}$. Is the problem convex? Does Slater's condition hold? Does strong duality hold?

   - minimize x subject to $x^2 \leq 1$
   - minimize x subject to $x^2 \leq 0$
   - minimize x subject to $|x| \leq 0$
   - minimize x subject to $f_1(x) \leq 0$, where

$$f_1(x) = \begin{cases} -x+2 & x \geq 1 \\ x & -1 \leq x \leq 1 \\ -x-2 & x \leq -1 \end{cases}$$

   - minimize x^3 subject to $-x+1 \leq 0$
   - minimize x^3 subject to $-x+1 \leq 0$ on domain $\mathbf{R}_+$