Convex Optimization 2025/26

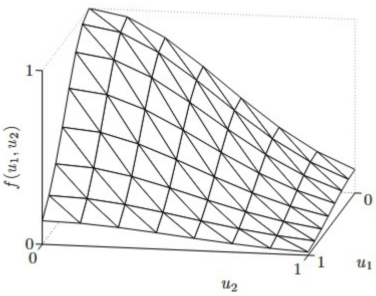
Practical session # 9

November 27, 2025

1. Mixed strategies for matrix games (BV Section 5.2.5)
   Example: Rock / Paper / Scissor.

2. Discussion: Find a convex problem that models the problem of fitting a function $f \in F$ to given data-points $u_1, \ldots, u_m \in \mathbf{R}^n$ and $t_1, \ldots, t_m$, where

   - F are polynomials of total degree $\leq d$
   - $n = 1$, and F are trigonometric polynomials of bounded degree
   - $n = 1$, and F are functions that are piece-wise linear between points $a_1 < a_2 < a_3 < \cdots < a_l$
   - $n > 1$ and F is piecewise linear on some fixed triangulation:

   Can we find a (quasi)-convex problem for fractions of polynomials of bounded degree?

   $$F = \{\frac{p(x)}{q(x)} \mid \deg(p) \leq k, \deg(q) \leq l, q(u_i) > 0 \text{ for } i = 1, \ldots, m\}$$

3. Show that for points $u_1, \ldots, u_k \in \mathbf{R}^n$ and values $t_1, t_2, \ldots, t_k \in \mathbf{R}$ there is a convex function $f : \mathbf{R}^n \to \mathbf{R}$ with $f(u_i) = t_i$ if and only if there are $g_1, \ldots, g_k \in \mathbf{R}^n$ such that $t_i \leq t_j + g_i^T(t_j - t_i)$ for all i, j.

4. Use Exercise (3) to state a quadratic program that is equivalent to the following problem:

$$\begin{array}{ll} \text{minimize} & \sum_{i=1}^{m} (t_i - f(u_i))^2 \\ \text{subject to} & f : \mathbf{R}^n \to \mathbf{R} \text{ is convex.} \end{array}$$