Modely rovnováhy nabídky a poptávky

Př: MODEL I - modifikace

$$D_t = \alpha + a_1 \underbrace{P_t}_{\text{cena}} + a_2 (P_t - P_{t-1}) \qquad S_t = \beta + b \cdot P_{t-1}$$

$\alpha, a_1, a_2 < 0$, $\beta, b > 0$, P_0 dáno, $t = 0, 1, 2, \dots$

$\in R$ $\quad \in R$

$$D_t = S_t$$

(1) $\alpha + a_1 P_t + a_2 (P_t - P_{t-1}) = \beta + b \cdot P_{t-1}$

$P_t = \bar{P} \;\forall t$ (2) $\alpha + a_1 \bar{P} + a_2 (\bar{P} - \bar{P}) = \beta + b \bar{P}$ $\ominus$

$$a_1 (P_t - \bar{P}) + a_2 (P_t - \bar{P}) - a_2 (P_{t-1} - \bar{P}) = b \cdot (P_{t-1} - \bar{P})$$

$$a_1 p_t + a_2 p_t - a_2 p_{t-1} = b\, p_{t-1}$$

$$(a_1 + a_2) p_t - (a_2 + b) p_{t-1} = 0 \qquad \text{HDR 1. řádu}$$

$$p_t - \frac{a_2 + b}{a_1 + a_2} p_{t-1} = 0$$

$$\lambda = \frac{a_2 + b}{a_1 + a_2} \Rightarrow p_t = C \cdot \left(\frac{a_2 + b}{a_1 + a_2}\right)^t \qquad p_0 = P_0 - \bar{P}$$

$$p_t = p_0 \left(\frac{a_2 + b}{a_1 + a_2}\right)^t$$

$$P_t - \bar{P} = \left(\frac{a_2 + b}{a_1 + a_2}\right)^t (P_0 - \bar{P})$$

a) $\left|\frac{a_2 + b}{a_1 + a_2}\right| < 1 \Rightarrow \lim\limits_{t \to \infty} P_t - \bar{P} = 0$

b) $= 1, > 0 \Rightarrow \lim\limits_{t \to \infty} P_t - \bar{P} = P_0 - \bar{P}$, je-li $-1 = \frac{a_2 + b}{a_1 + a_2} < 0$, nekonverguje

c) $\frac{a_2 + b}{a_1 + a_2} < -1 \Rightarrow \lim\limits_{t \to \infty} (P_{2t} - \bar{P}) = \infty$, $\lim\limits_{t \to \infty} (P_{2t+1} - \bar{P}) = -\infty$

$\frac{a_2 + b}{a_1 + a_2} > 1 \Rightarrow \lim\limits_{t \to \infty} P_t - \bar{P} = \infty$

Model II: $D_t = \alpha + a P_t$, $\alpha, a < 0$

$S_t = \beta + b(P_{t-1} - \rho \Delta P_{t-1})$, $\beta, b > 0$, $\rho \in (0,1)$, $\Delta P_{t-1} = P_{t-1} - P_{t-2}$

pro pokles ceny oč. růst a opačně

P_0, P_1 dáno

$D_t = S_t$

$\alpha + a P_t = \beta + b(P_{t-1} - \rho \Delta P_{t-1})$, pokud položíme $\bar{P}_t = \bar{P} \ \forall t$

$p_t = P_t - \bar{P}_t$ odečtu (2) od (1)

$\alpha + a\bar{P} = \beta + b(\bar{P} - \rho(\bar{P} - \bar{P}))$

$D = \alpha + aP$, $S = \beta + bP$ $\Big\}$ $\bar{P} = \frac{\alpha - \beta}{b - a}$

$a \cdot p_t = b p_{t-1} - b p_{t-1}\rho + b p_{t-2}\rho$

$p_t - \frac{b}{a}(1-\rho) p_{t-1} - \frac{b}{a}\rho\, p_{t-2} = 0$ HDR 2. řádu

$P_0, P_1 \Rightarrow$ jedn. řešení: $\lambda^2 - \frac{b}{a}(1-\rho)\lambda - \frac{b}{a}\rho = 0$ $D = \left[\frac{b}{a}(1-\rho)\right]^2 + 4 \cdot \left(\frac{b}{a}\rho\right)$

* $\exists \alpha \in (0,1): \left|\frac{b}{a}\right| \le \alpha \Rightarrow |\lambda_1| < 1$ a $|\lambda_2| < 1$ a $\lambda_1 \neq \lambda_2$

$p_t = C_1(\lambda_1)^t + C_2(\lambda_2)^t$

$\lim_{t\to\infty} p_t = 0$, $\lim_{t\to\infty} P_t = \bar{P}$

Obecněji: * 2 reálné kořeny:

$p_t = C_1(\lambda_1)^t + C_2(\lambda_2)^t$

+ kombinace
- * $|\lambda_1|, |\lambda_2| < 1$ ... $\lim_{t\to\infty} p_t = 0$
- * $|\lambda_1| = |\lambda_2| = 1$ $\xrightarrow{\text{osciluje}}$ $C_1 + C_2$; $\lambda_1 = -\lambda_2$ NEKONV.
- * $|\lambda_1| < 1$, $\lambda_2 > 1$ $\Rightarrow +\infty$ ...

* 1 dvojnásobný reálný

* 2 komplexní

Model III (modifikace - klasický Samuelson)

$$D(t) = \alpha + a P(t) + a_1 \frac{dP}{dt} \quad, \alpha \in R, a_1 < 0, a < 0$$

$$S(t) = \beta + b P(t) \quad, \beta \in R, b > 0 \quad, t \geq 0$$

$$b = 0$$

$$D(t) = S(t)$$

$$\alpha + a P(t) + a_1 \frac{dP}{dt} = \beta$$ odečtu $$\bar{P}(t) = \bar{P} \quad \forall t \geq 0$$

$$\alpha + a \bar{P} + a_1 \frac{d\bar{P}}{dt} = \beta \qquad d\bar{P}/dt = 0$$

$$a[P(t) - \bar{P}] + a_1 \left[\frac{dP}{dt} - \frac{d\bar{P}}{dt}\right] = 0$$

$$a\, p(t) + a_1 \frac{dp(t)}{dt} = 0$$ H Dif. Rce 1. řádu

$$\frac{dp(t)}{dt} = -\frac{a}{a_1} p(t)$$

$$\int \frac{p'(t)}{p(t)} dt = -\int \frac{a}{a_1} dt$$

$$\ln |p(t)| = -\frac{a}{a_1} t + C$$

$$p(t) = \underbrace{\exp(C)}_{p_0 = P(0) - \bar{P}} \exp\left(-\frac{a}{a_1} t\right)$$

$$\lim_{t \to \infty} p(t) = 0$$ model konvergentní

při $b > 0, a < 0, a_1 > 0$ $\lim_{t \to \infty} |p(t)| = +\infty$.