$$r(x) = c \iff -\frac{u''(x)}{u'(x)} = \phi \, c$$

$$u''(x) + c u'(x) = 0$$

$$-c = \frac{d}{dx} \ln u'(x)$$

$$-cx + a = \ln u'(x) \quad , \quad a \in \mathbb{R}$$

$$e^{-cx+a} = u'(x)$$

$$\underline{u(x) = -\frac{1}{c} e^{-cx+a} + d \quad , \quad a, d \in \mathbb{R}}$$

$$c > 0 \Rightarrow \text{rizikově averze}$$

2, klesající ARA (DARA)

speciálně: hyperbolicky klesající (HARA)

$$r(x) = \frac{1}{ax+b} \quad \text{pro } ax+b > 0$$

$$u(x) = \frac{c}{a-1} (ax+b)^{\frac{a-1}{a}} + d \quad \text{pro } a \neq 0, \, a \neq 1, \, ax+b > 0$$

$$= c \ln(x+b) + d \quad \text{pro } a = 1, \, x+b > 0$$

$$= -c \, e^{-\frac{1}{b}x} + d \quad \text{pro } b > 0, \, a = 0, \, c \geq 0, \, d \in \mathbb{R}$$

Zpět k mírám rizika:

THEORY OF COHERENT MEASURES

Measures of risk assingns a real number to any random variable L (loss):

- variance: $var(L) = \mathbb{E}(L - \mathbb{E}L)^2 \Rightarrow$ standard deviation: $sd(L) = (\mathbb{E}(L - \mathbb{E}L)^2)^{1/2}$

- semivariance: $r_s(L) = \mathbb{E}[\max(0, L - \mathbb{E}L)^2]$

- mean absolute deviation: $r_a(L) = \mathbb{E}|L - \mathbb{E}L|$

- mean absolute semideviation: $r_{as}(L) = \mathbb{E}[\max(0, L - \mathbb{E}L)]$

- value at risk: $VaR_\alpha(L) = \inf\{I \in \mathbb{R}, P(I \geq L) \leq 1 - \alpha\}$

- conditional value at risk: $CVaR_\alpha(L) = \inf\{a \in \mathbb{R}, a + \frac{1}{1-\alpha}\mathbb{E}[\max(0, L-a)]\}$

$$= \beta \mathbb{E}[L \mid L > VaR_\alpha(L)] + (1-\beta)VaR_\alpha(L); \quad \beta \in [0,1]$$

? Risk measures

- What are the "reasonable" properties that should have all "good" risk measures?

- Which of the considered measures has the properties?

- Is it possible to generalize a very well-known and popular standard deviation (variance)?

- What is the dual expression of measures with these properties?

? Multiobjective optimization