Model of a Joint Stock Company and related topics

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Overview

- Stochastic models and their parameter estimation.

- The model of Joint-Stock company

- The presentation of the current research in this field
- Potential of the model and further research

Crucial Problems needed to be solved to perform decent SP models

- Parameter estimation of the processes used to fit the data
- Choice of statistical distribution of the random innovations:
  1. Stable distribution (Natural conditions, not panacea)
  2. Hyperbolic, NIG, Variance Gamma
  3. IG, general jump processes
  4. Elliptical distributions.

Importance of Fit

- It determines how useful the theoretical results are
- Bad estimators: Hill for the stable distribution
- If the distribution is heavy tailed, then without knowledge of the particular features of the distributions we cannot get reliable estimates.

Stable GARCH(1, 1)-model and its parameter estimation

$$r_n = \mu + \sigma_n \varepsilon_n$$

$$\sigma_n^{\delta} = a_0 + a_1 \left| r_{n-1} - \mu \right|^{\delta} + b_1 \sigma_{n-1}^{\delta}$$

$$\varepsilon_n \sim S_\alpha(\sigma, 0, 0)$$

$$\sigma, \sigma_0, \delta, \mu, \alpha, a_0, a_1, b_1$$

$$r_n = \ln \frac{S_{n+1}}{S_n} \qquad \frac{r_n - \mu}{\sigma_n} = \varepsilon_n$$

Parameter Estimation

- $$\max_{\theta} L(\theta)$$

  $$L(\theta) = \ln\left(\prod_{j=1}^{n} \sigma_j^{-1} f_{\alpha,\sigma}\left(\frac{r_j - \mu}{\sigma_j}\right)\right)$$

  $$\theta = (\alpha, \sigma, \sigma_0, \delta, \mu, a_0, a_1, b_1)^T$$

  $$r_j = \ln\left(\frac{S_{j+1}}{S_j}\right),$$

  $$\sigma_m^{\delta} = a_0 + a_1 | r_{m-1} - \mu |^{\delta} + b_1 \sigma_{m-1}^{\delta},$$

Approximation of J

$$J(x,\alpha) = \frac{\partial \ln(f(x,\alpha))}{\partial \alpha} \qquad \hat{\alpha}_{ML} = \left\{ \alpha : \sum_{i=1}^{n} J\left(X_i, \alpha\right) = 0 \right\}$$

$$J_k(x,\alpha) = \sum_{j=0}^{k} a_j \exp(i \cdot t_j \cdot x) = \sum_{j=0}^{k} a_j \cos(t_j \cdot x) + i \sum_{j=0}^{k} a_j \sin(t_j \cdot x)$$

$$t = \left(t_1, t_2, \ldots, t_k\right)^T \qquad t_j = \frac{j}{k}$$

$$F(t,x) = \left(1, 2\cos(t_1 x), 2\cos(t_2 x), 2\cos(t_3 x) \ldots 2\cos(t_k x)\right)^T$$

$$J_k(x,\alpha) = \left(f(\alpha,k)\right)^T \cdot F(t,x)$$

Modified Form:

$$\sum_{j=1}^{n} \tilde{J}_{15}\left(\frac{r_j-\mu}{\sigma_j},\alpha\right)=0$$

$$r_j = \ln\left(\frac{S_{j+1}}{S_j}\right),$$

$$\sigma_m^{\delta} = a_0 + a_1 \,|\, r_{m-1}-\mu \,|^{\delta} + b_1\sigma_{m-1}^{\delta},$$

$$h_m = \mu + \sigma_m \varepsilon_m, \quad m = 1, 2, \ldots, n$$

$$s.t.$$

$$\alpha \in (1,\ 2],$$

$$\sigma, \sigma_0 > 0,$$

$$a_0 + a_1 + b_1 < 1,$$

$$1 \le \delta < \alpha,\ \ \mu \in \square\,.$$

$$D_{n,n} < D_{n,n}^{*}(\beta)$$

Results

Table 1. Estimates of the parameters of the GARCH(1,1)-model.

Estimator	Value
$\left\lvert \sum_{j=1}^{n} \tilde{J}_{15}(\hat{\varepsilon}_j, \hat{\alpha}) \right\rvert$	0.020010
$\hat{\alpha}$	1.780000
$\hat{\sigma}$	0.049980
$\hat{\sigma}_0$	0.005640
$\hat{\delta}$	1.334383
$\hat{\mu}$	0.000800
$\hat{a}_0$	0.068328
$\hat{a}_1$	0.039289
$\hat{b}_1$	0.149827

Source: Author's computations

Comparison of the distributions

- Jump Processes
- Hyperbolic distributions
- Univariate Stable
- Multivariate Stable
- Operator Stable

The problem

- 1) Dividends, Debt, Emision

$$\max\{P,E\} \qquad U(E+, dividends+, P+, emission-, debt-...)$$

$$s.t.\ n\cdot P/E \in (a,b); \qquad s.t.\ constraint$$

- 2) E – New business for debt/emission/Assets
- 3) P – Dividend policy/PR/Ad
- 4) P is a jump process. Jumps caused by dividends. Economic theory

High Values of nP/E

- 1) Signals that the stocks are overestimated.
- 2) Profits don’t justify high values of shares.
- 3) Big risks.
- 4) Cannot afford: dividends and emission.
- 5) Debt to open new business, enlarge production
- 6) Size effect is not related to big IT companies due to the fall of their stock prices preceded by huge values of the P/E ratio.

WolframAlpha computational knowledge engine

p/e ratio google

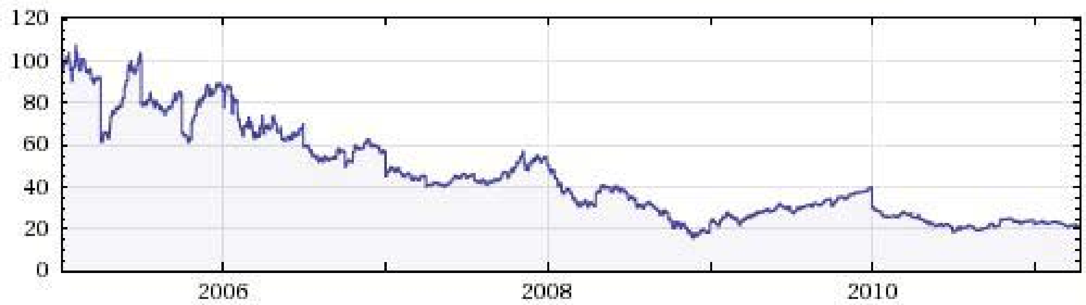
Input interpretation:

Google	p/e ratio

Result:

21.39 (Tuesday, April 12, 2011)

History: Last 10 years



(from Jan 2, 2005 to Apr 11, 2011)

minimum	average	maximum
15.95	45.52	107.5
(quarter ending Nov 24, 2008)		(quarter ending Feb 3, 2005)

Input interpretation:

Apple	p/e ratio

Result:

18.26 (Tuesday , April 12, 2011)

History: Last 10 years

(from Apr 16, 2001 to Apr 11, 2011)

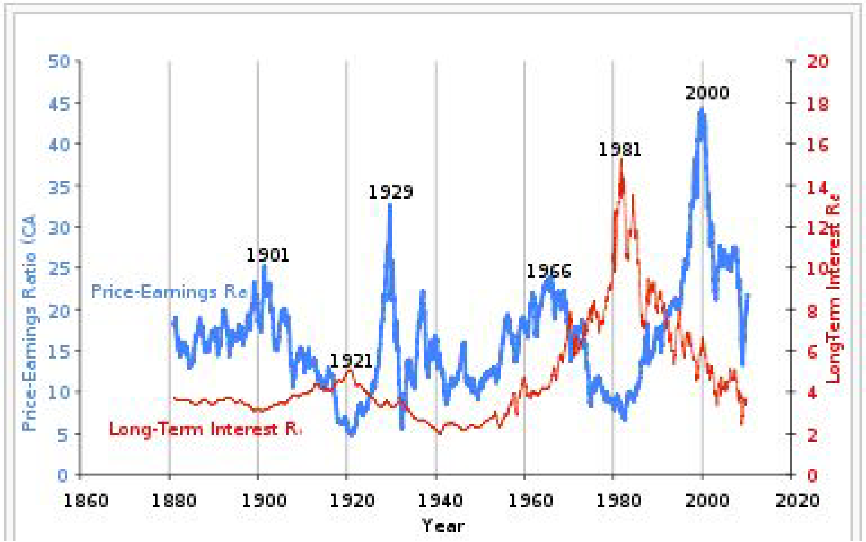
minimum	average	maximum
14.24 (quarter ending Jan 20, 2009)	47.09	305.4 (quarter ending Feb 18, 2003)

P/E

Maximum: Mining company Talvivaara: 99000

From 1920-1990: between 10 and 20

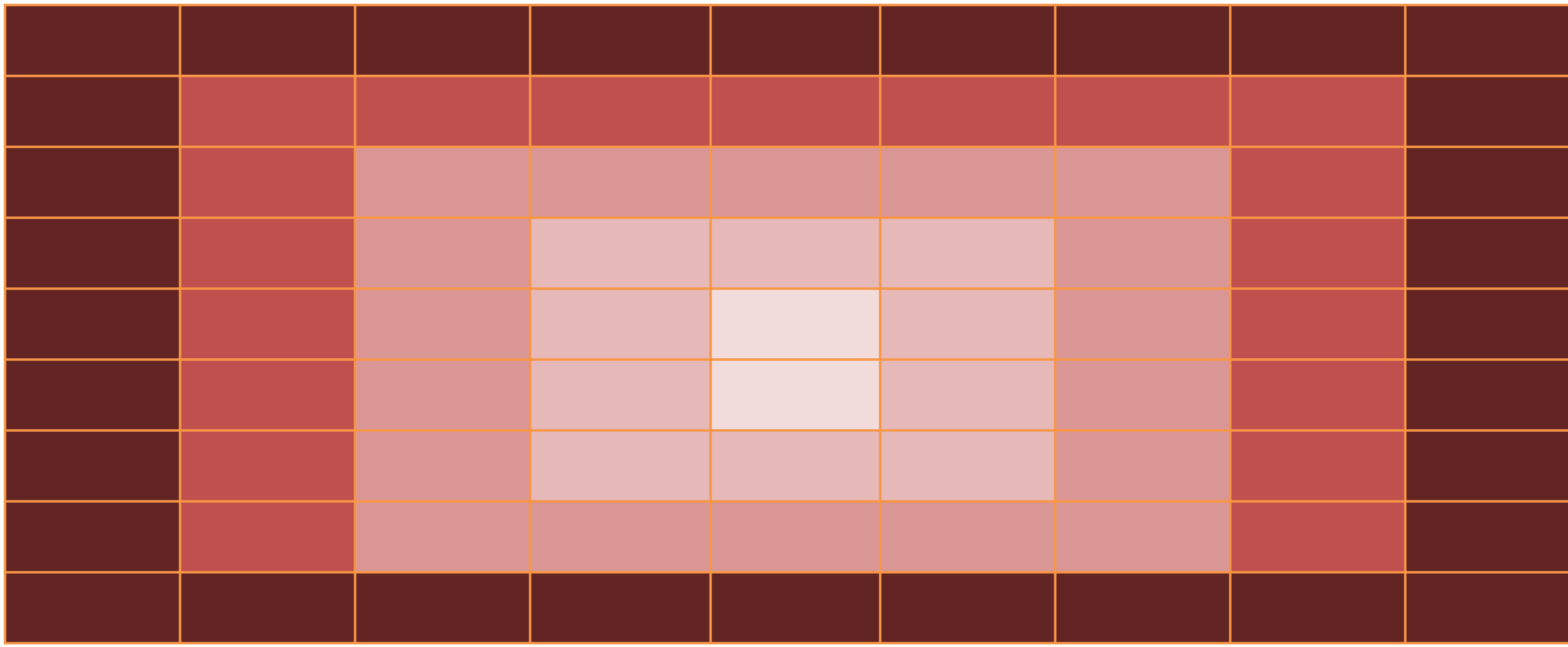
Dotcom: 46.50 in 2001



Utility and Dividends

- Dividends increase utility. The CEO’s of stock companies are shareholders and the source of their income is: profit, dividends, capital returns of the stocks. Therefore P/E too small is not desired for them.
- The decisions about dividends, debt and emission are based on some stopping times entailed from utility function.

The territorial structure of business. The closer to the center, the higher demand and fixed costs



Here it means that the lighter the color of the cells, the higher corresponding parameters.

The specification of the Problem

Actions in Multistage:
1) Paying dividends +
2) Expansion +
3) Contraction -
4) Debt -
5) Emission -

The may overlap

$\max_{D^*,E^*,E,P,d,e,f} U(P, D^*, E^*, E, d, e, f)$
$s.t.$

$E_t = \sum_{j=1}^{K} \sum_{k=t-m}^{t} \tilde{I}(j)\varepsilon_{j,k}$
$\varepsilon_{j,k} = F_j(\bar{x}_j, \hat{D}_j)$, $\bar{x}_j = \text{argmin}_{x\geq 0} \{f_j(x) := E[F_j(x, D_j)]\}$
$F_j(x, D_j) = c \cdot x + b \cdot [D_j - x]_+ + h \cdot [x - D_j]_+ - FC_j$
$P_t \sim GBM$ or GLP
"$P/E \in [a, b]$", $P/E = n \cdot P_t/E_t$, n is the number of stocks
$P\left(P/E \in [a, b]\right) \geq 1 - \alpha$ or $\mathcal{E}\left(P/E\right) \in [a, b]$
$D_t^* \leq x\%\text{Wealth}_t$ $E_t^* \leq y\%\text{Wealth}_t$ $E_t^* + D_t^* \leq z\%\text{Wealth}_t$
$d_t = I(P/E > b) f_1(P_t +, -||-) + I(E_t < 0) f_2(|E_t| +, -||-)$
e_t emission. Last resort The decisions must be based on stopping times of P_t, E_t, P/E and other parameters.

In the news vendor problem, $\bar{x}_j = H^{-1}\left[\frac{b-c}{b+h}\right]$ where $P(D_j \leq x) = H(x)$

Characteristics of the Objective Function

Random character of price

Random character of demand

$$U(P,D,E,d,e,f,FC)=\frac{P^{1-p_0}}{1-p_0}+\frac{D^{1-p_1}}{1-p_1}+\frac{E^{1-p_2}}{1-p_2}-\frac{d^{1-p_3}}{1-p_3}-\frac{e^{1-p_4}}{1-p_4}-g(f)-G(p/e)$$

Random character of IR

$$Profit = E + E_{Porfolio} + e - d - FC$$

$$\max\left\{U(P,D,E,d,e,f,FC)=\frac{P^{1-p_0}}{1-p_0}+\frac{D^{1-p_1}}{1-p_1}+\frac{E^{1-p_2}}{1-p_2}-\frac{d^{1-p_3}}{1-p_3}-\frac{e^{1-p_4}}{1-p_4}-g(f)-G(p/e)\right\}$$

$$E_t=\sum_{j=1}^{K}\sum_{k=t-m}^{t}\tilde{I}(j)\cdot\varepsilon_{jk};\qquad \varepsilon_{jk}=c\cdot\overline{x}+b\cdot\left[D_j-\overline{x}\right]_+ +h\cdot\left[\overline{x}-D_j\right]_+ -FC_j\qquad \overline{x}=H^{-1}\left(\frac{b-c}{b+h}\right)$$

$$"n\cdot P_t/E_t\in(a,b)"$$

$$P_t=P_0\exp\left(\left(\mu-\frac{\sigma^2}{2}\right)t+\sigma\cdot W_t+J_t(D)\right),\ \ J_t(D)\geq 0$$

$$d_t=I(P/E>b)F_1(P/E;...)+F_2(E_t-,D(j)+,FC(j)-,TotalDebt_t-)$$

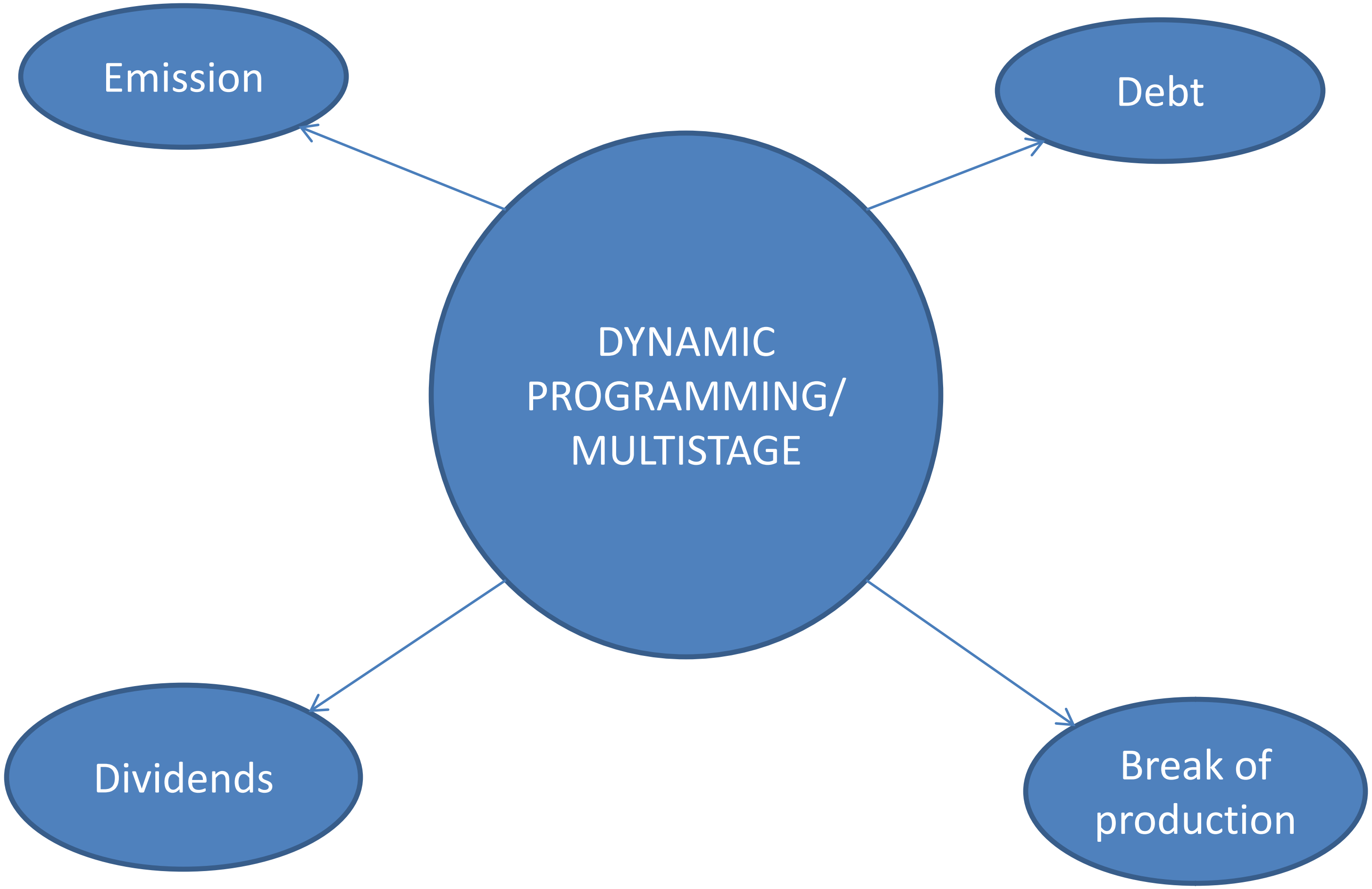
f_j qualitative variable. Decision to finish/quit production at location j, depends on profitability and fixed costs at location j. Yes/No

$$g:(f_1,f_2,f_3,...,f_K)\to R^+\cup\{0\},\ \ g(No,No,...,No)=0$$

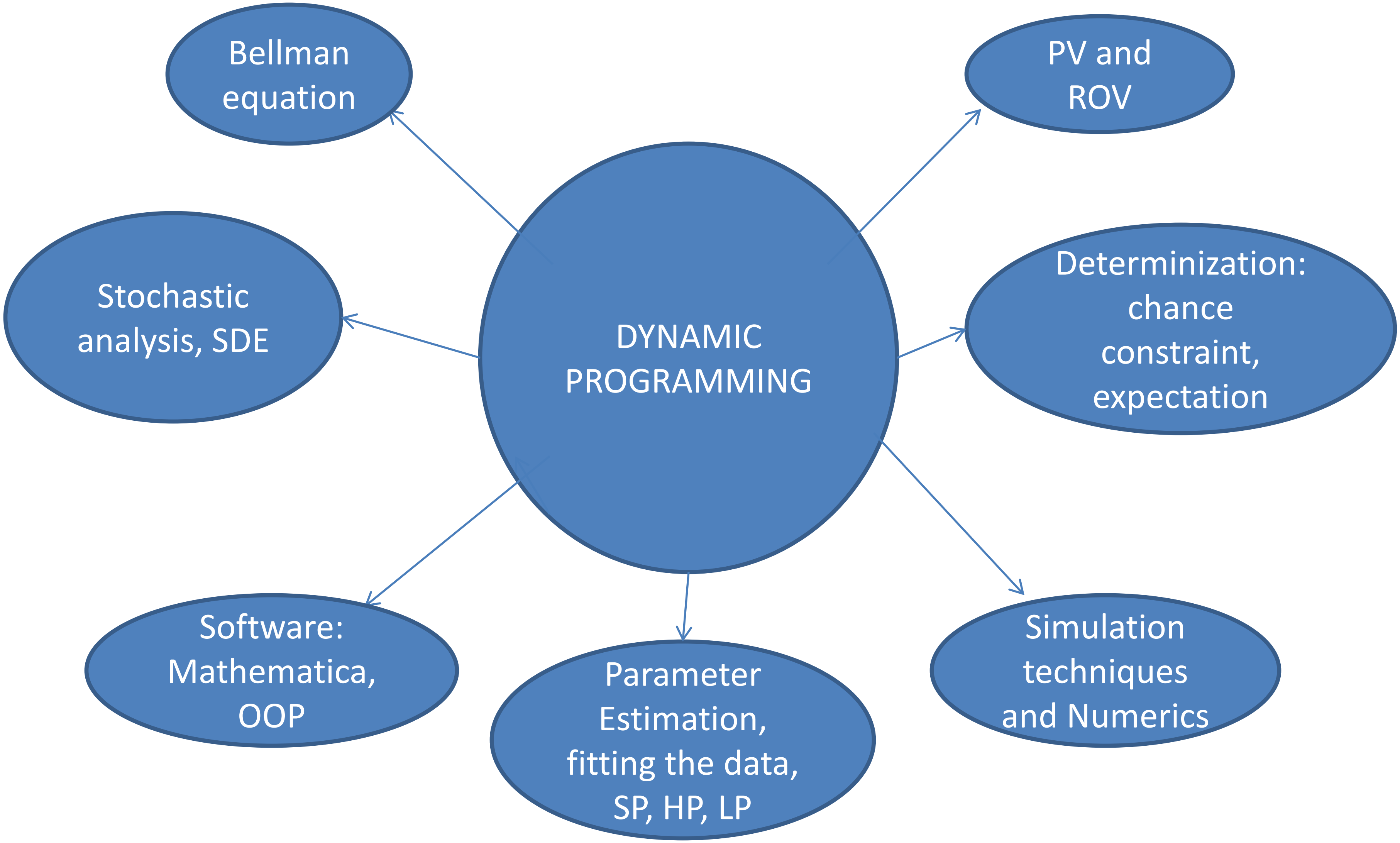
$G(P/E)=0\ \ if\ \ P/E<b$ Otherwise it is a non-decreasing function of P/E, P/E is a shorthand for $n\cdot P_t/E_t$

$$P_t=P_0\exp\left(\left(\mu-\frac{\sigma^2}{2}\right)t+\sigma\cdot W_t+J_t(D)+J_t*(P/E)\right),\ \ J_t(D)\geq 0,\ \ J_t*(P/E)\leq 0$$

Actions: 3 decisions of quantitative character, 1 decision of qualitative character



Tools needed for the project



Current Work

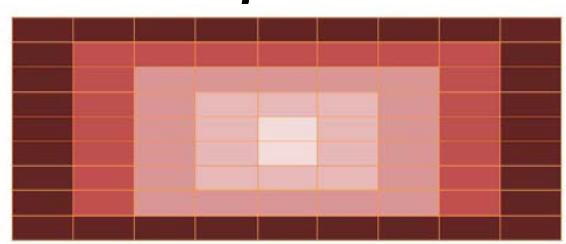
- Dividends and Expansion
- Demand is ruled by a distribution with a.s. positive values.
- Utility function: power
- Software: Mathematica
- Aim: to write a flexible program ready for further improvements and adjustments.
- Application: study expansion and dividend policy of a firm (Joint-stock company)

Description of the Model

- We observe the business for m days and then decide about dividends and expansion
- Utility function

$$\text{Maximize}\left[\left\{\frac{(aa\,x)^{1-p}}{1-p}+\frac{(cc\,(Assets-x-y))^{1-p1}}{1-p1}+\frac{(bb\,y)^{1-p2}}{1-p2},\right.\right.$$
$$x\geq 0,\ y\geq 0,\ x\leq 0.05\,Assets,\ y\leq 0.12\,Assets,$$
$$\left.\left.x+y\leq 0.13\,Assets\right\},\ \{x,\ y\}\right]$$

- Distribution of the demand: Weibullian. Because it is positive-valued.
- The problem: news vendor is a joint stock company selling news



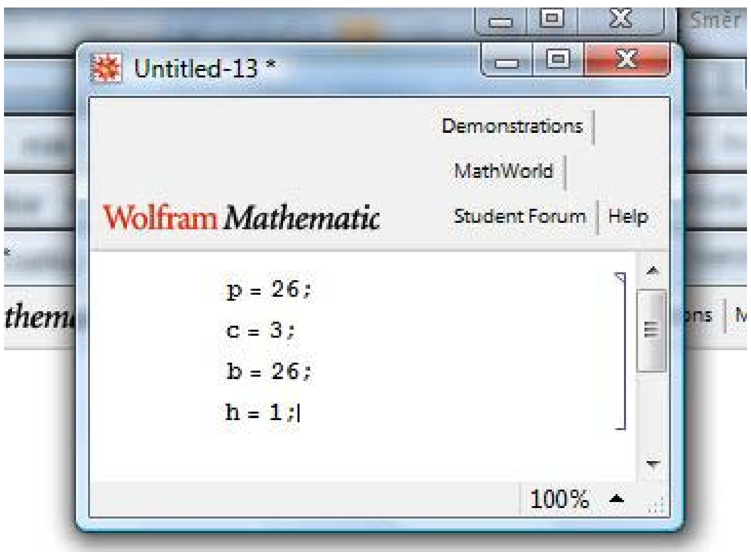
Description of the Model

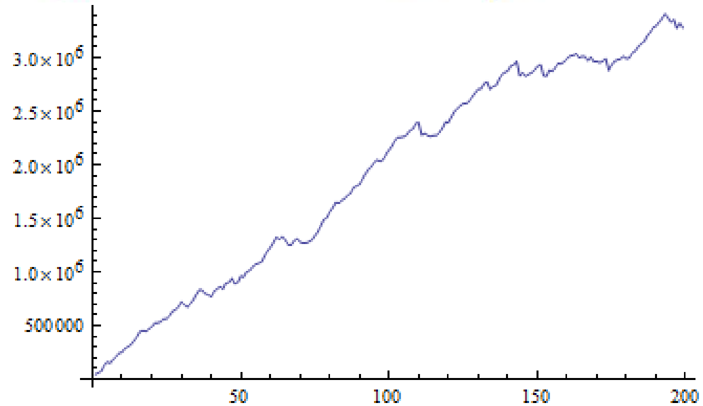
- At any stage the investor decides if she will expand or pay dividends. It may be both based on the parameters of the utility function. If she decides to expand, she will have to pay for the NB.
- The dividends are paid based on the given wealth.
- The more attractive business, the higher costs of the new business and the higher fixed costs.
- If Revenue-Costs+Wealth<0 then the investor goes bankrupt.

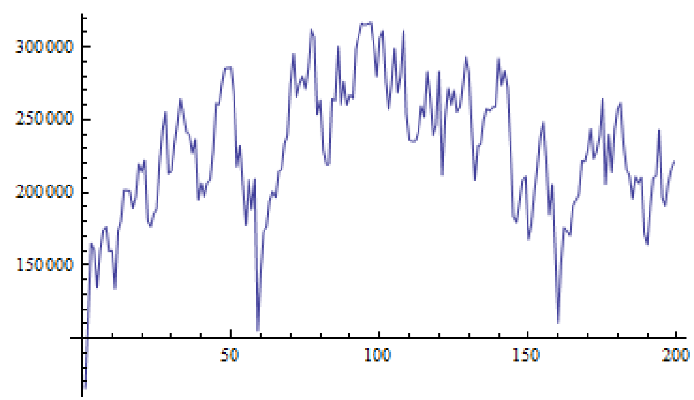
Results for Different Parameters of the Utility Function

$$\text{Revenues} - \text{Costs} = p\bar{x} - \left(c \cdot \bar{x} + b \cdot [\bar{x} - \hat{D}]_+ + h \cdot [\hat{D} - \bar{x}]_+\right) - FC_j$$

$$U(x,y) = \frac{(a \cdot x)^{1-p}}{1-p} + \frac{(c \cdot (\text{Wealth} - x - y))^{1-p_1}}{1-p_1} + \frac{(b \cdot x)^{1-p_2}}{1-p_2}$$







Consequences of the CLT

$$E_t = \sum_{j=1}^{K} \sum_{k=t-m}^{t} \tilde{I}(j)\varepsilon_{j,k}$$

A sum of many summands. If we assume that the variance is finite, then it can be approximated by the normal distribution.
Otherwise, it may be a stable distribution.
If $E_t \sim S_\alpha(\sigma, 1, \mu)$ with $\alpha < 1$ then $P(E_t > 0) = 0$.
It will be easier to explore P/E. If α could equal 0.5, we would deal with the Levy distribution and we would need to explore the properties of the process

$$P_t/E_t = \exp\left(\mu t + \sigma L_t\right)/E_t = N^2 \cdot \exp\left(\mu t + \sigma L_t\right), \quad N \sim N(0,1)$$

But such process would be too much volatile. In this situation α should be close to 1.
Problems and challenges:

1. if α is close to 1 then the estimates are not so reliable.
2. An undesired situation is to get to a sum of the stable distributions with different α

General formula:

$$P_t/E_t = \exp\left(\mu t + \sigma L_t\right)/E_t = \exp\left(\mu t + \sigma L_t\right)/S, \quad S \sim S_\alpha(\sigma, \beta, \mu)$$

Practical applications and outcomes

Taking the data of stock prices and earnings,	Studying correlations and dependences of crushes and high values of P/E on real data	Use of the data and their analyses to adjust our dynamic programming model to reality
Generalization of the random process that rule the stock prices. Lévy processes with different increments: Normal, Stable, IG, NIG, Variance gamma, BM & Poisson	Determination of the optimal utility function. We will start with power utility.	The symphony of stochastic analysis and programming with features from economic theory makes the model less vulnerable to Lucas critique.
Black swanery: if the model is more or less adjusted to the reality, then lots of surprises may pop out: dividend policy, etc.	Possible objectives: survive as long as possible, obtain as high utility for the top management as possible, highest profit. It depends on the settings: parameters and functions.	Reminiscence of the past: Agent-based model. It is hard to make it decent enough to produce all the features of the real economy. Too much complicated

Theoretical background

All the actions of the dynamic programming can be based on stopping times	Approximation of the problem by simpler problems may lead to tractability and reasonable theoretical results	Using qualitative variables leads to mixed integer programming.
Production and Portfolio, General motors. Future research. Value-at-risk or CVaR constraints. Measures of risk	Production vs. Portfolio: under which conditions a firm may prefer profits from portfolio to profits from production	CVaR constraints in GARCH, ARMA-GARCH setup. Monte Carlo estimates are not reliable due to heavy tails. Stable/hyperbolic
Valuation of American options.	SAA estimators	Variance reduction techniques

Possible extensions of the model

- Include the second/third.. agent/firm. Another point of view how to explore profitability.

Agent 1 $G_{1,1}$	Agent 2 $G_{1,2}$
Agent 1 Agent 2 share the demand	Agent 2 $G_{2,2}$

$$G_{2,1}, Agent_1 = U(0, G_{2,1}),\ \ Agent_2 = G_{2,1} - Agent_1$$

- Competition environment.

Potential of such a model

- Estimation of the fair price of stocks by modeling the dividend policy. The distribution of time of dividends may be determined from the empirical data but such an approach is less vulnerable to the LC.
- Having fitted the distributions properly and having set the parameters, this model can be used for risk management.
- Other features were already mentioned in the previous slides.

Further development

- Portfolio analysis with Operator stable distributions.
- The tail indexes can be estimated from the univariate margins. The spectral measure determines the dependence structure between margins. E.g. copulas
- Simulations of OSD.

Literature

- Glasserman P. Monte Carlo Methods in Financial Engineering, Stochastic Modelling and Applied Probability, 2003
- Bibby B. M., Sorensen M.: Hyperbolic Processes in Finance, Handbook of Heavy Tailed Distributions in finance, Rachev eds.
- Shapiro A., Dencheva D.,Ruszchynski A., Lectures on Stochastic Programming Modeling and Theory, 2009