

Homework 2 – Integer linear programming and VaR

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COMPUTATIONAL ASPECTS OF OPTIMIZATION

Investment problem with VaR

Solve a simple investment problem

$$\begin{aligned} \min_{x_i} \quad & \text{VaR}_\alpha \left(- \sum_{i=1}^n x_i R_i \right) \\ \text{s.t.} \quad & \mathbb{E} \left[\sum_{i=1}^n x_i R_i \right] \geq r_0, \\ & \sum_{i=1}^n x_i = 1, \quad x_i \geq 0, \end{aligned}$$

where we consider n assets with random rate of return R_i . The first constraint ensures minimal expected return r_0 , x_i are (nonnegative) portfolio weights which sum to one.

Value at Risk (VaR)

Portfolio optimization problem:

$$\begin{aligned} \min_{z, x} z \\ P \left(- \sum_{i=1}^n R_i x_i \leq z \right) &\geq \alpha, \\ \sum_{i=1}^n \mathbb{E}[R_i] \cdot x_i &\geq r_0, \\ \sum_{i=1}^n x_i = 1, \quad x_i &\geq 0, \end{aligned}$$

where R_i is random rate of return of i -th asset and minimal expected return r_0 is selected in such way that the problem is feasible.

Homework 2

- 1 Rewrite the VaR minimization problem under a finite discrete distribution as a mixed-integer LP problem (using big- M).
- 2 Use the dataset from preliminary task. Use Python and algebraic language. You can start with the prepared template.
- 3 Consider $\alpha = 0.95$ and run the problem for different 11 values $r_0 \in \{\min_i \bar{R}_i, \dots, \max_i \bar{R}_i\}$.
- 4 Plot the optimal values VaR_α against the corresponding values of r_0 .