

Homework 3 – Nonlinear programming in portfolio optimization

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COMPUTATIONAL ASPECTS OF OPTIMIZATION

Mean–variance–skewness portfolio optimization

Consider n assets with random rates of return denoted by R_i , with $\mathbb{E}|R_i|^3 < \infty$ and define the corresponding covariance matrix C and skewness tensor S elementwise as

$$C_{jk} := \mathbb{E}(R_j - \mathbb{E}R_j)(R_k - \mathbb{E}R_k),$$

$$S_{jkl} := \mathbb{E}(R_j - \mathbb{E}R_j)(R_k - \mathbb{E}R_k)(R_l - \mathbb{E}R_l).$$

Employ the aggregate function approach of multiobjective optimization with aggregation parameter $c > 0$

$$\begin{aligned} & \text{minimize} \quad \sum_{j=1}^n \sum_{k=1}^n C_{jk} x_j x_k - c \sum_{j=1}^n \sum_{k=1}^n \sum_{l=1}^n S_{jkl} x_j x_k x_l \\ & \text{subject to} \quad \sum_{i=1}^n \mathbb{E}[R_i] \cdot x_i \geq r_0, \\ & \quad \quad \quad \sum_{i=1}^n x_i = 1, \quad x_i \geq 0. \end{aligned} \tag{1}$$

Homework 3

- 1 Use the returns data from preliminary task to estimate the mean vector, variance matrix and skewness tensor.
- 2 Solve the mean–variance ($c = 0$) and the mean–variance–skewness¹ ($c = 0.1$) problems.
- 3 Solve the problems for different 11 values $r_0 \in \{\min_i \bar{R}_i, \dots, \max_i \bar{R}_i\}$.
- 4 Plot the optimal values against the corresponding values of r_0 .
- 5 Use GAMS software. For the plot, use probably another software - read the results from a saved output file. Try different solvers, check if the results vary, comment on this in the homework.

¹The problem is nonconvex in general.

- ① Download and install GAMS software. Request a free demo license.
- ② When testing it, I did have a problem with the free demo license, the online form did not work. I wrote them an email and they responded in few hours with the license file.
- ③ In GAMS, in Model Libraries → Practical Financial Problems, you can find example programs that will help you get started and understand the structure of the code. A short template is also prepared.