2nd notes on Problem 2

I think that PA^- (or any extension by a finite number of IND axioms) is inadequate to formalize the construction outlined in the 1st notes on the problem. But we may try to isolate what it is we need.

Recall the definition of relation $P(x, y)$ in the 1st notes: it expresses that y is a power of 2 and it occurs in the unique expression of x as a sum of powers of 2. Using it define new relation with a suggestive symbol:

$$t \in x \iff \exists y \leq x,\ P(x, y) \wedge y = 2^t$$

where $y = 2^t$ is defined using the $|\ldots|$ function: $|y| = t + 1$. In other words, if $x = 2^{i_1} + \ldots + 2^{i_k}$, with $i_1 > \ldots > i_k$, we interpret x as the set $\{i_1, \ldots, i_k\}$.

What we need for the construction in the 1st notes to go through is a *comprehension scheme*:

$$\text{(CA)} \qquad \forall x \exists z \leq x \forall t \leq |x|,\ t \in z \equiv \psi(t)$$

where ψ is any Δ_0-formula. We can expresses the value $f(x)$ of the map defined in the 1st notes as the set defined by:

$$\{t \leq |x| \mid Q(x, 2^t\}$$

or writing Q explicitly

$$\{t - 1 \mid t \in x \wedge 2^t \notin J\} \cup \{t \mid t \in x \wedge 2^t \in J\}\ .$$

Note that CA follows from IND (see top of p.2 in the 1st notes) but **CA is all we need to run that argument**.

It is a good exercise to solve also - using CA - the case of WPHP talking about maps from n^2 to n (i.e. the second part of Problem 2).