The B-validity of the hypothesis of the CC axiom

On p.105 it is stated that we may assume w.l.o.g. that

$$[\![\exists y \forall x\, \varphi(x) \le y]\!] = 1_B.$$

This is seen as follows.

(1) Assume $[\![\exists y \forall x\, \varphi(x) \le y]\!] = b = U/_{[P=0]}$, where $U \in \mathcal{A}$. By the earlier construction during the verification of the validity of the AC axiom there is $y_0 \in R$ s.t.:

(a) $b = [\![\forall x\, \varphi(x) \le y_0]\!]$

(cf. also top of p. 105).

(2) Define now $\varphi' \in R^R$ s.t. for all $\xi \in R$:

$$\varphi'(\xi)(\omega) := \varphi(\xi)(\omega), \text{ if } \omega \in U$$

$$:= y_0(\omega), \text{ if } \omega \notin U.$$

As $U \in \mathcal{A}$, $\varphi'(\xi)$ is indeed ~~measurable~~ $\in R$.

(3) Claim: $[\![\forall x\, \varphi'(x) \le y_0]\!] = 1_B$.

Proof: Because (1)(a), for all $\xi \in R$:

(2)

$\{\omega \in \Omega \mid \varphi'(\xi)(\omega) \leq \eta_0(\omega)\} \supseteq U \in [p = c]$

and outside U $\varphi'(\xi)(\omega)$ equals to $\eta_0(\omega)$.

So $[\![\forall x\, \varphi'(x) \leq \eta_0]\!] = b \vee \neg b = 1_B$.

Claim.

(4) It remains to show that:

$\underline{[\![\rho \text{ is the l.u.b. for } \varphi']\!] = 1_B} \Rightarrow [\![\exists \text{ l.u.b. for } \varphi]\!] \geq b.$

But $[\![\varphi' = \varphi]\!] \geq b$ and hence $\Rightarrow$ implies

$[\![\rho \text{ is the l.u.b. for } \varphi]\!] \geq b \wedge 1_B = b.$

q.e.d.