MAF - Konzultace 23.10.2020 10^{00}

$$2! \cdot 4! \dots (2n)! > \left((n+1)!\right)^n$$

$$\frac{(2k+2)!}{(k+1)!} = \frac{(2k+2)\cdot(2k+1)\cdot 2k \dots (k+2)\cancel{(k+1)k \dots 3\cdot 2\cdot 1}}{\cancel{(k+1)\cdot k \dots 3\cdot 2\cdot 1}}$$

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$$\lim_{x\to 1} \frac{x+x^2+\dots+x^n-n}{x-1} = \lim_{x\to 1}\left(\frac{x-1}{x-1}+\frac{x^2-1}{x-1}+\dots+\frac{x^n-1}{x-1}\right)$$

$$\frac{x-1}{x-1} = 1 \to 1, \quad \frac{x^2-1}{x-1} = x+1 \to 2, \quad \dots \to n-1, \quad \frac{x^n-1}{x-1} = \underbrace{x^{n-1}+x^{n-2}+\dots+1}_{\to n}$$

$$= 1+2+\dots+(n-1)+n = \frac{n(n+1)}{2}$$

$$(x^n-1) = (x-1)\cdot(x^{n-1}+\dots+x+1)$$

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③ $\lim\limits_{x\to 2}\left(\frac{1}{x^2-2x}-\frac{x}{x^2-4}\right)=\lim\limits_{x\to 2}\left(\frac{1}{x(x-2)}-\frac{x}{(x+2)(x-2)}\right)=$

$=\lim\limits_{x\to 2}\frac{x+2-x^2}{x(x+2)(x-2)}=\lim\limits_{x\to 2}\frac{-\cancel{(x-2)}(x+1)}{x(x+2)\cancel{(x-2)}}=$

dosaď $x=2$

$-x^2+x+2=-(x^2-x-2)$

$=-(x-2)(x+1)$

Cv2 ⑧ $(2n)!<2^{2n}(n!)^2 \quad \forall n\in\mathbb{N}$

1) $n=1 \quad L=2!=2\ ,\ P=2^2\cdot 1!=4$ ✓

2) $n\longrightarrow n+1 \qquad (2n+2)!\overset{?}{<}2^{2n+2}\cdot((n+1)!)^2$

$(2n)!(2n+1)(2n+2)\overset{?}{<}2^{2n+2}\cdot((n+1)!)^2$

$(IP)\Rightarrow \quad <\cancel{2^{2n}}\cdot\cancel{(n!)^2}(2n+1)(2n+2)\le 2^{\cancel{2n}+2}\cdot\cancel{((n+1)!)^2}$ $(n+1)^2$

Chci

$(2n+1)\cancel{(2n+2)}\le(2n+2)\cancel{(2n+2)}$ ✓

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(11) $n^{n+1} > (n+1)^n \quad , \quad n \geq 3$

1) $n=3$ $\quad L = 3^4 = 81$ ✓

$\quad P = 4^3 = 64$

2) $n \mapsto n+1$

$$(n+1)^{n+2} \overset{?}{>} (n+2)^{n+1}$$

$$\Updownarrow \quad (n+1)^{n+1} \cdot (n+1) \overset{?}{>} (n+2)^{n+1}$$

$$n+1 \overset{?}{>} \left(\frac{n+2}{n+1}\right)^{n+1} = \left(1+\frac{1}{n+1}\right)^{n+1}$$

IP : $\quad n^{n+1} > (n+1)^n$

$$\Updownarrow \quad n > \left(\frac{n+1}{n}\right)^n = \left(1+\frac{1}{n}\right)^n > \left(1+\frac{1}{n+1}\right)^n \quad / \cdot \left(1+\frac{1}{n+1}\right)$$

$$\left(n+\underbrace{\frac{n}{n+1}}_{<1}\right) > \left(1+\frac{1}{n+1}\right)^{n+1}$$

QED

$n+1 >$

3)

D. Pokorný:

① Dokažte, že $7 \mid (5^{2n+1} + 2^{2n+1})$ pro $\forall n \in \mathbb{N}_0$

Bez indukce : $a^{2n+1} + b^{2n+1} = \underbrace{(a+b)}_{(5+2) = 7}(a^{2n} - a^{2n-1}b + \dots - ab^{2n-1} + b^{2n})$

Indukcí : 1) $n = 0$: $5^1 + 2^1 = 7$ ✓

2) $5^{2n+3} + 2^{2n+3} = 5^2 \cdot (5^{2n+1}) + 2^2 \cdot 2^{2n+1} =$

$= \underbrace{5^2 (5^{2n+1} + 2^{2n+1})}_{\text{(IP) } 7 \mid} \underbrace{- 5^2 \cdot 2^{2n+1} + 2^2 \cdot 2^{2n+1}}_{(4-25) \cdot 2^{2n+1}}$

$(4-25) = -21$, $7 \mid 21$

Př z písemky:

Dokažte $\forall n \in \mathbb{N}$: $n(n^2+5)$ je dělitelné šesti.

1) $n=1$: $1\cdot(1^2+5)=6$ ✓

2) $n \to n+1$: $(n+1)\left((n+1)^2+5\right) = (n+1)\left(n^2+2n+\overbrace{1+5}^{=6}\right) =$

$$= \underbrace{n\cdot(n^2+5)}_{(IP) \Rightarrow 6|} + \underbrace{n(2n+1) + (n^2+2n+6)}_{= 3n^2+3n+6 \text{ je děl. } 6 \Leftrightarrow}$$

$\Leftrightarrow 3\underbrace{n(n+1)}_{\text{vždy sudé ?}}$ je —"—

$\forall n \in \mathbb{N}$:

$$\frac{1}{1\cdot2\cdot3} + \frac{1}{2\cdot3\cdot4} + \dots + \frac{1}{n(n+1)(n+2)} = \frac{n(n+3)}{4(n+1)(n+2)}$$

Dokažte indukcí

17 $\lim\limits_{x\to 0} \dfrac{\sqrt{1+x} - \sqrt[3]{1-x}}{\sqrt[3]{1+x} - \sqrt{1-x}}$

Trik: převedete na $\sqrt[6]{\ldots}$

$$(1+x)^{\frac{1}{2}} = \left(\left(1+x\right)^{\frac{1}{6}}\right)^{3}$$