

Cvičenie MAF, 29.9.2020, 900

operace v \mathbb{C} :

$$z = a+bi, w = c+di$$

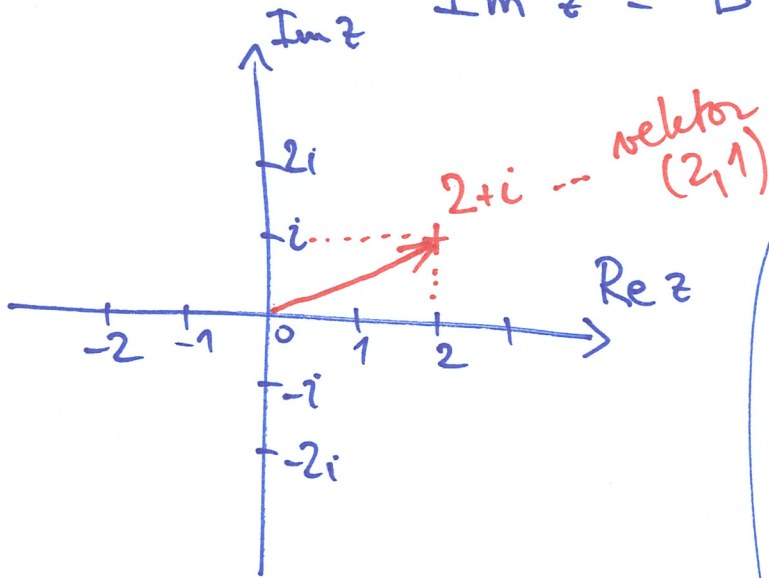
Komplexní čísla: \mathbb{C}

$$\sqrt{-1} = ? \quad i \notin \mathbb{R}$$

$$\mathbb{C} = \{a+bi, a, b \in \mathbb{R}\}$$

$$z = a+bi \Rightarrow \operatorname{Re} z = a$$

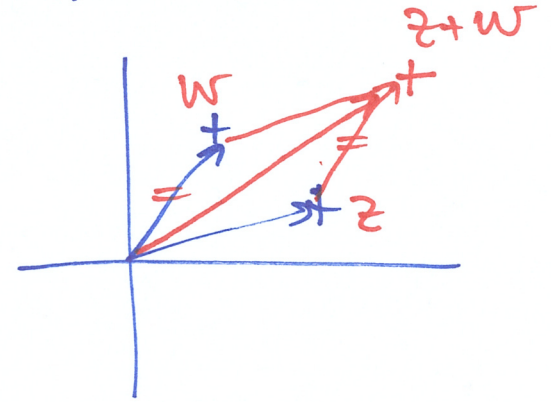
$$\operatorname{Im} z = b$$



Gaussova rovina

$$z+w = (a+c) + (b+d)i$$

odpovídá
vektorům v \mathbb{R}^2



$$\begin{aligned} z \cdot w &= (a+bi) \cdot (c+di) = \\ &= ac + adi + bci + bdi^2 \\ &= (ac - bd) + (ad + bc)i \end{aligned}$$

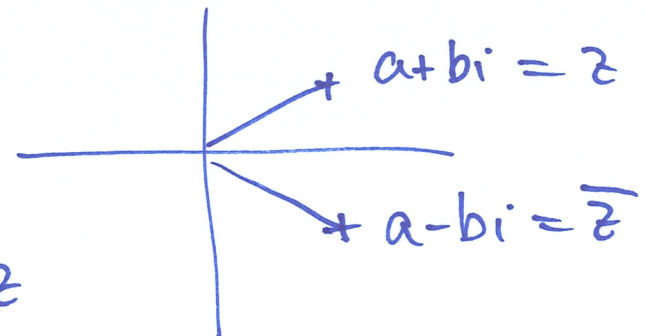
$$i^2 = -1$$

konjugované komplexní čísla $z \mapsto \bar{z}$

$$z = a+bi$$

$$\bar{z} = a-bi$$

$$z \in \mathbb{R} \Leftrightarrow \bar{z} = z$$



①

velikost (absol. hodnota) : $|z| \in \langle 0, +\infty \rangle$

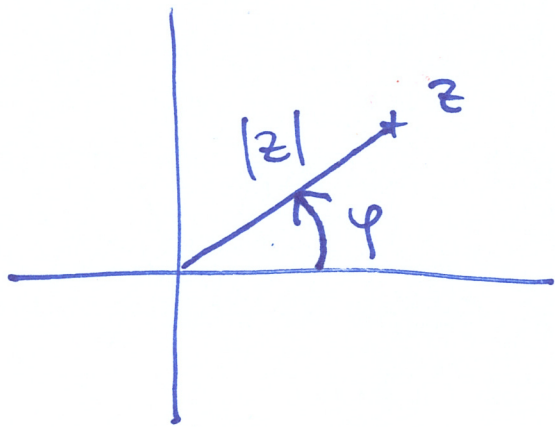
$$|z|^2 = z \cdot \bar{z} = (a+bi) \cdot (a-bi) = \\ = a^2 - (bi)^2 = a^2 + b^2 \geq 0$$

odpovídá velikosti vektoru v \mathbb{R}^2

$$\frac{1}{z} = z^{-1} = \frac{\bar{z}}{|z|^2} \quad \dots \text{dělení}$$

($z \neq 0$)

goniometrický zápis komplex. čísla



$$z = |z| \cdot (\cos \varphi + i \sin \varphi)$$

pro $z \neq 0$

$\varphi = \text{Arg } z$ (argument
komp. čísla z)

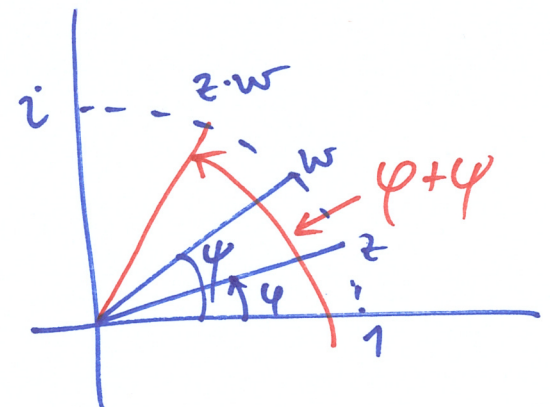
násobení v goniom. zápisu:

$$z = |z| \cdot (\cos \varphi + i \sin \varphi)$$

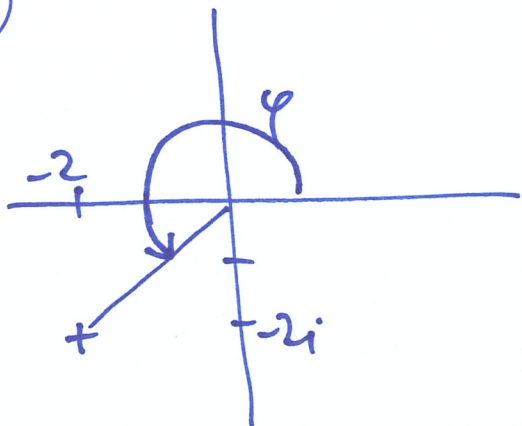
$$w = |w| \cdot (\cos \psi + i \sin \psi)$$

$$z \cdot w = |z| \cdot |w| \cdot (\cos(\varphi + \psi) + \\ i \sin(\varphi + \psi))$$

pro $|z| = |w| = 1$ (komplexní
jednotky)



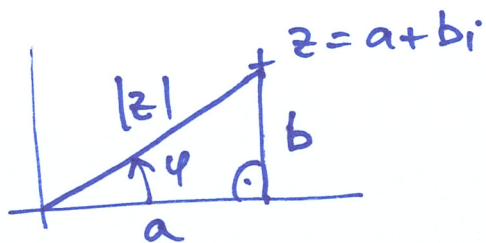
2a



$$\arg z = \varphi \in \langle 0, 2\pi \rangle$$

7

3f



$$\cos \varphi_1 = \frac{a_1}{|z_1|}, \quad \sin \varphi_1 = \frac{b_1}{|z_1|}$$

$$\cos \varphi_2 = \frac{a_2}{|z_2|}, \quad \sin \varphi_2 = \frac{b_2}{|z_2|}$$

$$z_1 \cdot z_2 = (a_1 a_2 - b_1 b_2) + (a_1 b_2 + a_2 b_1) i =$$

$$= |z_1| |z_2| (\cos \varphi_1 \cos \varphi_2 -$$

Negace kvantifikatoru:

$$\text{non } (\exists x \in \mathbb{R} : \cos x = \sqrt{1 - \sin^2 x}) \equiv$$

$$\equiv \forall x \in \mathbb{R} : \cos x \neq \sqrt{1 - \sin^2 x}$$

doplnujici dotaz:

plati:

$$\forall x \in \mathbb{R} : \cos x = \sqrt{1 - \sin^2 x} \quad ?$$

non

$$\exists x \in \mathbb{R} : \cos x \neq \sqrt{1 - \sin^2 x}$$

NEPLATI

2

8a) platí $\forall a \in \mathbb{R} \exists \varepsilon > 0 \exists \alpha \in \mathbb{R} : \forall x \in \mathbb{R} : x \in (a, a+2) \Leftrightarrow |x - (a+1)| < 1$

pro dané a volíme $\varepsilon = 2$
 $\alpha = a+1$

8b) neplatí
pro $\varepsilon = 3, \alpha = a+1,5 \Rightarrow |x - \alpha| \geq 1$ pro některá $x \in (a, a+3)$
např. $x = a+2,6$

10) $f(M_1) \setminus f(M_2) \subset f(M_1 \setminus M_2)$ Dokažte.
vezmu libov. $y \in$ a chci ukázat, že $y \in f(M_1 \setminus M_2)$

$f(M_1)$

PRÍSTĚ