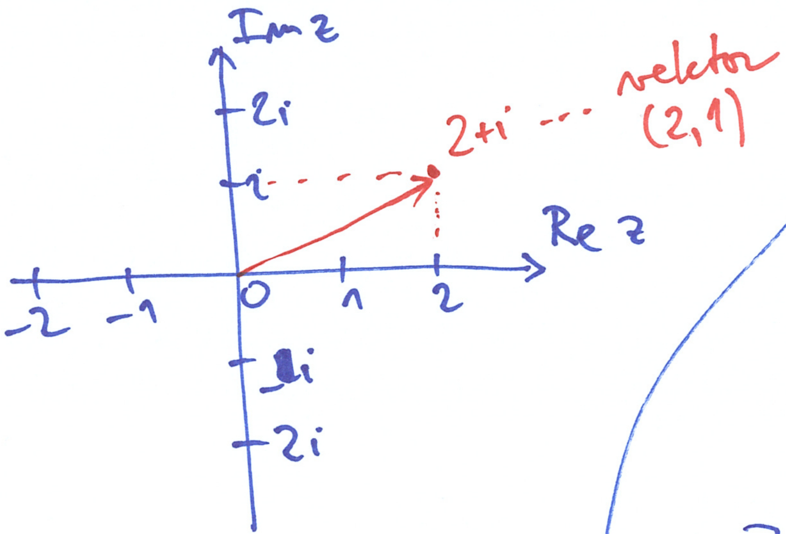
Cvičení MAF, 29.9.2020, 12:20

Komplexní čísla $\mathbb{C}$

$\sqrt{-1} = ?$, $i \in \mathbb{C}$, $i \notin \mathbb{R}$

$$\mathbb{C} = \{a+bi;\ a, b \in \mathbb{R}\}$$

$$z = a+bi \Rightarrow \begin{aligned} \operatorname{Re} z &= a \\ \operatorname{Im} z &= b \end{aligned}$$



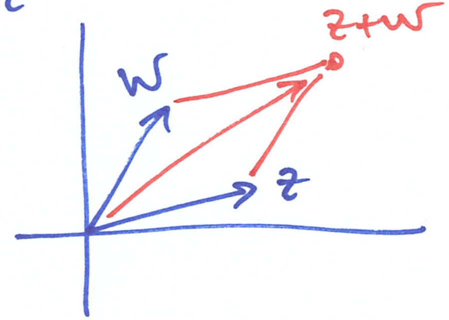
Gaussova rovina

operace v $\mathbb{C}$:

$z = a+bi$, $w = c+di$

$$z+w = (a+c) + (b+d)i$$

odpovídá sčítání vektorů v $\mathbb{R}^2$



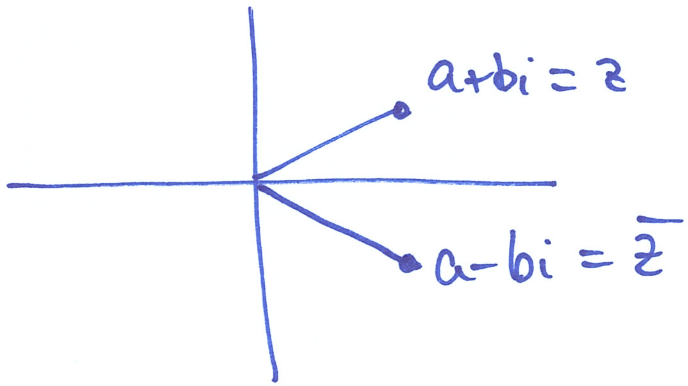
$$z \cdot w = (a+bi) \cdot (c+di) =$$
$$= ac + adi + bci + bdi^2 =$$
$$= (ac-bd) + (bc+ad)i$$

$$\boxed{i^2 = -1}$$

komplexní sdružení $z \longleftrightarrow \overline{z}$

$z = a+bi$

$\overline{z} = a-bi$



$$z \in \mathbb{R} \Leftrightarrow \overline{z} = z$$

velikost (absolutní hodnota): $|z| \in \langle 0, +\infty)$

$$|z|^2 = z \cdot \overline{z} = (a+bi)\cdot(a-bi) =$$
$$= a^2 - (bi)^2 = a^2 + b^2 \geq 0$$

odpovídá velikosti vektoru v $\mathbb{R}^2$

$$\frac{1}{z} = z^{-1} = \frac{\overline{z}}{z\cdot\overline{z}} = \frac{\overline{z}}{|z|^2} \quad \text{... dělení}$$

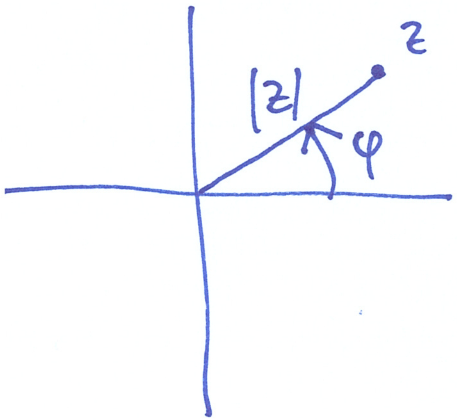
$(z \neq 0)$

násobení v goniom. zápisu:

$$z = |z|\cdot(\cos\varphi + i\sin\varphi)$$
$$w = |w|\cdot(\cos\psi + i\sin\psi)$$
$$z\cdot w = |z|\cdot|w|\cdot\big(\cos(\varphi+\psi) + i\sin(\varphi+\psi)\big)$$

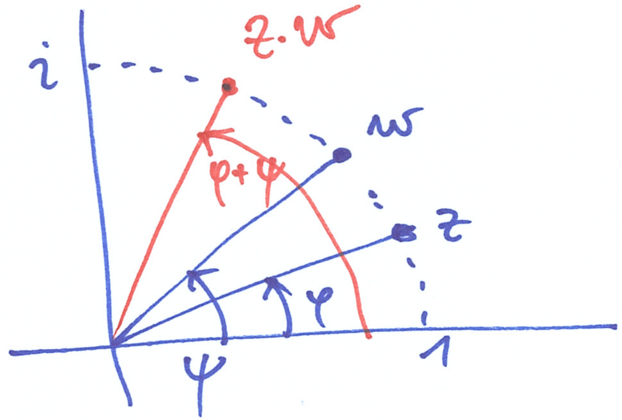
pro $|z| = |w| = 1$ (komplexní jednotky)

goniometrický zápis kompl. čísla



$$z = |z|\cdot(\cos\varphi + i\sin\varphi)$$
$$z \neq 0$$
$\varphi = \arg z$ (argument kompl. čísla z)

$$\varphi \in \langle 0, 2\pi)$$



1a) $\frac{2}{1-3i} = 2\cdot\frac{1}{1-3i} = 2\cdot\frac{1+3i}{1+9} = \frac{2}{10}(1+3i)$

1b) $1+i\sqrt{3} = |2|\cdot(\cos 60^\circ + i\sin 60^\circ)$, $z^3 = 8\cdot(\underbrace{\cos 180^\circ}_{=-1} + i\underbrace{\sin 180^\circ}_{=0}) = \underline{\underline{-8}}$

3e) $|z_1\cdot z_2| = |z_1|\cdot|z_2| \quad /(\ )^2$

$z_1 = a+bi$
$z_2 = c+di$

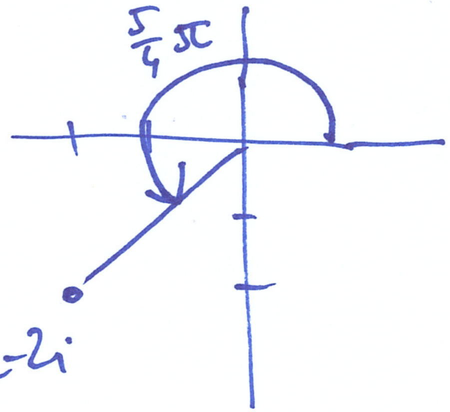
$z_1\cdot z_2 = (ac-bd)+(ad+bc)i$

$|z_1\cdot z_2|^2 = (ac-bd)^2+(ad+bc)^2$

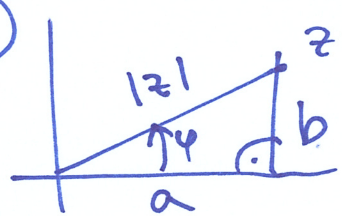
$=$

$\left.\begin{array}{l}|z_1|^2 = a^2+b^2\\ |z_2|^2 = c^2+d^2\end{array}\right\}$ součin

2a)



3f)



$\cos\varphi_1 = \frac{a_1}{|z_1|}$, $\sin\varphi_1 = \frac{b_1}{|z_1|}$

$\cos\varphi_2 = \frac{a_2}{|z_2|}$, $\sin\varphi_2 = \frac{b_2}{|z_2|}$

$z_1\cdot z_2 = (a_1\cdot a_2 - b_1\cdot b_2)+(a_1b_2+a_2b_1)i =$

$= |z_1|\cdot|z_2|\big((\cos\varphi_1\cdot\cos\varphi_2 - \sin\varphi_1\cdot\sin\varphi_2)+(\ldots)i\big)$

3 ~~4~~

7) ✓ $\exists x \in \mathbb{R} : \cos x = \sqrt{1-\sin^2 x}$

negace

$\forall x \in \mathbb{R} : \cos x \neq \sqrt{1-\sin^2 x}$

$\forall x \in \mathbb{R} : \cos x = \sqrt{1-\sin^2 x}$

negace

✓ $\exists x \in \mathbb{R} : \quad \neq$

✓ $\forall x \in \langle -\frac{\pi}{2}, \frac{\pi}{2} \rangle : \cos x = \sqrt{\quad}$ platí

8a) $\alpha := a, \quad \varepsilon = 0{,}5$

$x \in (a, a+0{,}5) \Rightarrow |x-a| < 1$

$\varepsilon = 2, \quad x \in (a, a+2) \Leftrightarrow |x-(a+1)| < 1$

$\alpha = a+1$

8b) $a = 2, \ \varepsilon = 1 \qquad x \in (2,3)$

$\alpha = 100 \qquad x \in (99, 101)$ $\not\Leftrightarrow$

10) zobrazení $f : X \to Y$

$x \mapsto f(x)$

$(x \in X)$

$M \subset X \Rightarrow f(M) =$

$= \{ f(m);\ m \in M \}$ ... obraz množ. M

Př: $f : \mathbb{Z} \to \mathbb{Z}$

$a \mapsto 2a$

$f(\mathbb{Z}) = \{ \text{sudá čísla} \}$

$f(\{1,2,3\}) = \{2,4,6\}$

$\underbrace{f(M_1) \setminus f(M_2)}_{y \in} \subset \underbrace{f(M_1 \setminus M_2)}_{y \in}$

$y \in \ \Longrightarrow \ y \in$

$\boxed{\text{PŘÍŠTĚ}}$