

Cvičení č. 7: Limity funkcí III

1. Vypočtěte limity:

(a) $\lim_{x \rightarrow 0} \frac{1 - \cos x \cos 2x \cos 3x}{1 - \cos x}$

(b) $\lim_{x \rightarrow \pi} \frac{\sin nx}{\sin mx}$, $n, m \in \mathbb{N}$

(c) $\lim_{x \rightarrow \frac{\pi}{4}} \operatorname{tg}(2x) \operatorname{tg}\left(\frac{\pi}{4} - x\right)$

(d) $\lim_{x \rightarrow 0} \frac{\sin(a+2x) - 2\sin(a+x) + \sin a}{x^2}$, $a \in \mathbb{R}$

(e) $\lim_{x \rightarrow 0} \frac{\operatorname{cotg}(a+2x) - 2\operatorname{cotg}(a+x) + \operatorname{cotg} a}{x^2}$, $a \in \mathbb{R}$, $\sin a \neq 0$

(f) $\lim_{x \rightarrow 0^+} \frac{\arccos(1-x)}{\sqrt{x}}$

(g) $\lim_{x \rightarrow 0^+} \frac{\frac{\pi}{2} - \arcsin \frac{1}{\sqrt{x^2+1}}}{x}$

(h) $\lim_{x \rightarrow 0} \frac{\ln(a+x) + \ln(a-x) - 2\ln a}{x^2}$, $a > 0$

(i) $\lim_{x \rightarrow 0} \frac{\ln(\operatorname{tg}(\frac{\pi}{4} + ax))}{\sin bx}$, $a, b \in \mathbb{R}$, $b \neq 0$

(j) $\lim_{x \rightarrow 0^+} \ln(x \ln a) \ln\left(\frac{\ln ax}{\ln \frac{x}{a}}\right)$, $a > 0$

(k) $\lim_{x \rightarrow 0} \frac{\ln(1+xe^x)}{\ln(x+\sqrt{1+x^2})}$

(l) $\lim_{x \rightarrow 1} (1-x) \log_x 2$

Řešení:

1. (a) 14; (b) $(-1)^{n+m} \frac{n}{m}$; (c) $\frac{1}{2}$; (d) $-\sin a$; (e) $\frac{2\cos a}{\sin^3 a}$; (f) $\sqrt{2}$; (g) 1; (h) $-\frac{1}{a^2}$; (i) $\frac{2a}{b}$; (j) $2 \ln a$; (k) 1; (l) $-\ln 2$.