

Mathematics for Economists I Problems 2

Graphs of simple functions

Draw precisely (using a ruler) the graph of the given function:

1. $4x - 2$

2. $-2x + 4$

3. $|x - 3|$

4. $2 - |x + 1|$

Points C, D are given in the plane. Find the equation of the line which passes through them, in the form $y = ax + b$.

5. $C = [2, 5], D = [4, 3]$

6. $C = [-2, -1], D = [7, 2]$

For the given quadratic function, find the intercepts with axes and the vertex of the parabola, draw the graph (by hand).

7. $4 + 3x - x^2$

8. $x^2 + 2x + 2$

9. Draw graphs of functions $x^2, x^2 - 3, (x - 2)^2, (x - 2)^2 - 3 = x^2 - 4x + 1$ into one common picture and observe how they are obtained by translation of the "basic parabola" x^2 .

10. Use the "completing the square" method on the function $x^2 + 6x + 2$ to find the vertex of the parabola. Compute roots in the usual way and draw the graph.

For the given linear fractional function find the intercepts with the axes, coordinates of the center of the hyperbola and the asymptotes, draw the graph (by hand).

11. $\frac{3x-1}{x+2}$

12. $\frac{x+2}{2x-1}$

Applications

13. You are deciding between two mobile phone plans. In the first plan (A), you pay a monthly fee of CZK 300 and CZK 2 per minute called. In the second plan (B), you pay a monthly fee of CZK 500, you get 60 minutes a month for free, and you pay CZK 1.50 for each additional minute you call. (For simplicity, we do not consider SMS or data charges.) Which plan is more advantageous depending on how many minutes you use per month?

Try to represent the solution graphically: the horizontal axis of the graph will represent used minutes, the vertical axis will represent the monthly price.

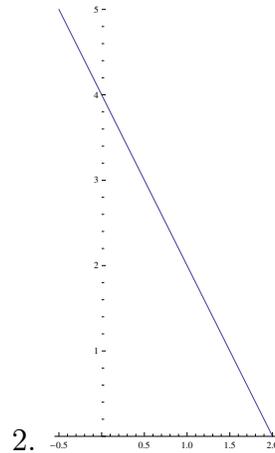
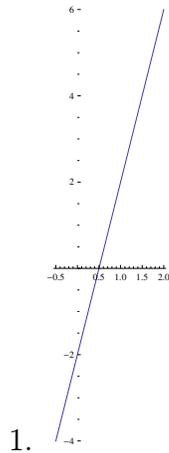
14. A consumer wants to buy a larger quantity of a certain type of product. He has two purchase options. A) He buys the goods in a store in the immediate vicinity of his home, where he pays CZK 27.50 for 1 kg of the goods. B) He drives his car to the manufacturer, where 1 kg of this product costs only CZK 19. However, he has to pay additional 340 CZK for the gas used. At what quantity of goods will it be financially

more advantageous for the consumer to buy directly from the manufacturer? Solve the problem numerically and graphically.

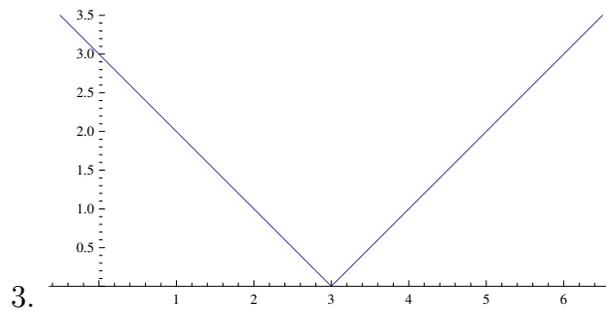
Solutions:

1. see picture.

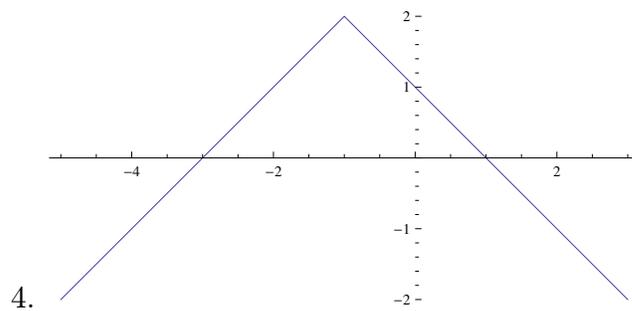
2. see picture.



3. see picture.



4. see picture.

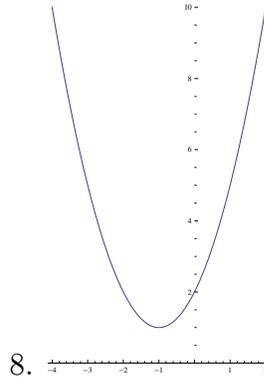
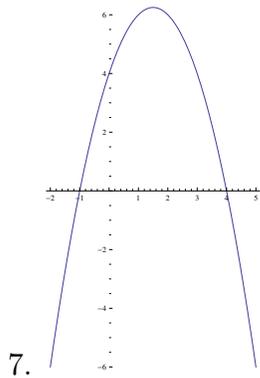


5. $y = -x + 7$.

6. $y = \frac{1}{3}x - \frac{1}{3}$.

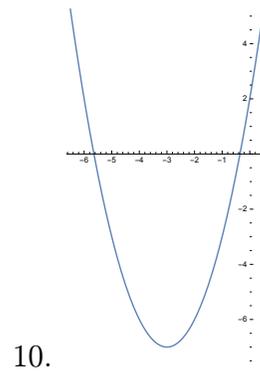
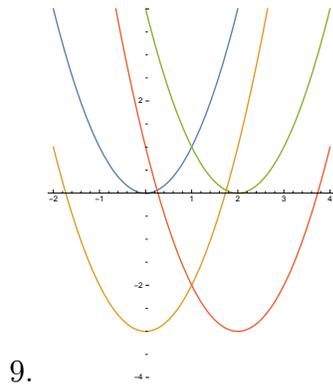
7. intercepts $[-1, 0]$, $[4, 0]$, $[0, 4]$, vertex $[\frac{3}{2}, \frac{25}{4}]$.

8. intercepts $[0, 2]$, vertex $[-1, 1]$.

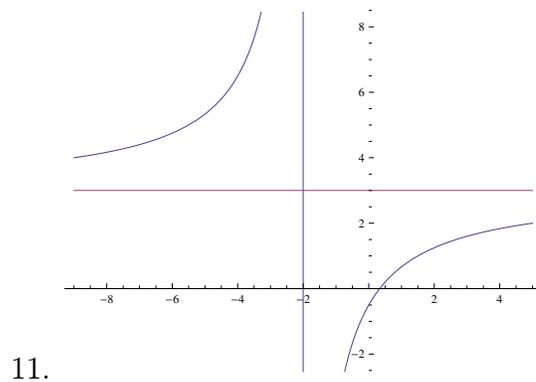


9. see picture.

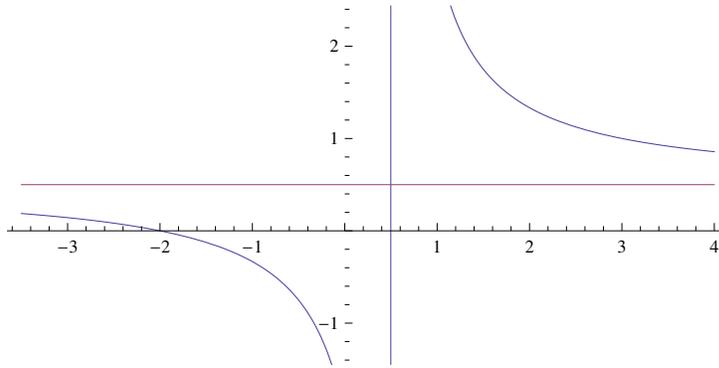
10. $f(x) = (x + 3)^2 - 7$.



11. intercepts $[1/3, 0]$, $[0, -1/2]$, center $[-2, 3]$, asymptotes $y = 3$, $x = -2$.



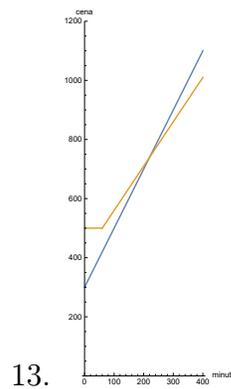
12. intercepts $[-2, 0], [0, -2]$, center $[1/2, 1/2]$, asymptotes $y = 1/2, x = 1/2$.



12.

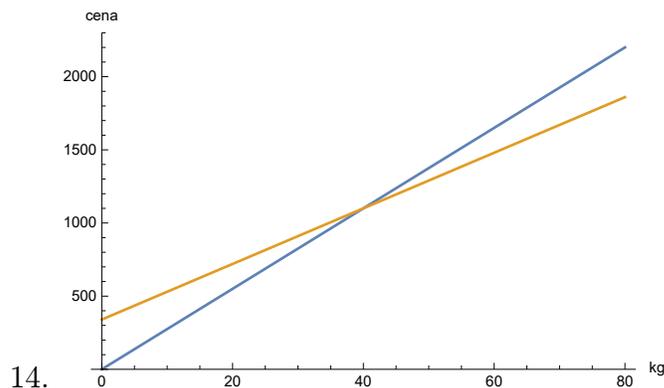
13. With 220 called minutes per month, you will pay the same in both plans – 740 CZK. If you call less, the lower plan (A) is more advantageous, if you call more, the higher plan (B) is more advantageous.

For plan A, the graph is a straight line (it is a linear function). For plan B, the graph is a broken line: if you call anything between 0 and 60 minutes, you pay a constant 500 CZK (corresponds to the horizontal line), and from the 60th minute on, the dependence on the number of minutes will have effect, from there the graph starts to rise linearly.



13.

14. At 40 kg, he will pay the same. At any higher quantity it is worth going to the manufacturer.



14.