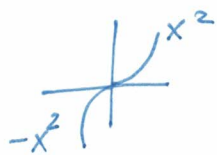


Společné derivace (z definice)

1. $f(x) = x|x|$



$$f'(0) = \lim_{h \rightarrow 0} \frac{(0+h)|0+h| - f(0)}{h} = \lim_{h \rightarrow 0} |h| = \text{VOLSA}$$

a) $\lim_{h \rightarrow 0} |h| = |0|$

b) $\lim_{h \rightarrow 0^+} h = 0$ ($= \lim_{h \rightarrow 0} h = 0$, pokud \exists)

$$\lim_{h \rightarrow 0^-} -h = -0 = 0$$
 ($= \lim_{h \rightarrow 0} -h = -0$, pokud \exists)

$$c) f'_+(0) = \lim_{h \rightarrow 0^+} \frac{(0+h)|0+h| - f(0)}{h} = \lim_{h \rightarrow 0^+} h = \lim_{h \rightarrow 0} h = 0$$

[než h > 0] $\left[\text{if } \exists \right]$

$$f'_-(0) = \lim_{h \rightarrow 0^-} \frac{(0+h)(-h) - 0}{h} = \lim_{h \rightarrow 0^-} (-h) = -\lim_{h \rightarrow 0} h = 0$$

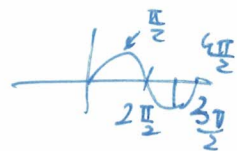
2. $f(x) = \begin{cases} x \sin \frac{1}{x} & x \neq 0 \\ 0 & x = 0 \end{cases}$

* Viž s. 1 [více o spojitosti této fce].

$$f'(0) = \lim_{h \rightarrow 0} \frac{h \sin \frac{1}{h} - f(0)}{h} = \lim_{h \rightarrow 0} \sin \frac{1}{h}$$

Neurčitosti $h = \frac{2}{\pi(2n+1)}$ $\sin\left(\frac{\pi(2n+1)}{2}\right) = (-1)^n$

$$h = \frac{2}{\pi 2n} \quad \sin\left(\frac{\pi 2n}{2}\right) = 0$$



3. $f(x) = \begin{cases} x^\alpha \sin \frac{1}{x} & x \neq 0 \\ 0 & x = 0 \end{cases}$ $\alpha > 1$

$$f'(0) = \lim_{h \rightarrow 0} \frac{h^\alpha \sin \frac{1}{h}}{h} = \lim_{h \rightarrow 0} h^{\alpha-1} \sin \frac{1}{h}$$

Lemma o poličkách:

$$0 \leq \left| h^{\alpha-1} \sin \frac{1}{h} \right| \leq |h^{\alpha-1}|$$

$$\downarrow \quad \downarrow \quad \alpha > 1$$

$$0 \quad 0$$

$$\Rightarrow \lim_{h \rightarrow 0} |h^{\alpha-1} \sin \frac{1}{h}| \stackrel{\&}{=} 0 \Rightarrow$$

$$\lim_{h \rightarrow 0} (h^{\alpha-1} \sin \frac{1}{h}) = 0$$

Je vůbec $f(x) = x \sin \frac{1}{x}$ spojité?

1'

$f(x)$ pro $x \neq 0$ jestložením spojité

Pro $x = 0$. Že spočítat limitu?

$$\begin{array}{ccc} 0 \leq |x \sin \frac{1}{x}| \leq |x| & & \\ \downarrow & & \downarrow x \rightarrow 0 \\ 0 & & 0 \end{array}$$

Dle lemma pnutí o palicích je $\lim_{x \rightarrow 0} |x \sin \frac{1}{x}| = 0$.

Plně $\lim_{x \rightarrow x_0} |f| = 0 \Rightarrow \lim_{x \rightarrow x_0} f = 0$ je $\lim_{x \rightarrow 0} (x \sin \frac{1}{x}) = 0$

limit. $\exists \wedge f(0) = 0$. Také f je spoj. i v 0.
ajerovna 0

$\lim_{x \rightarrow x_0} |f| = c \Rightarrow \lim_{x \rightarrow x_0} f = c$ uplatí. $f(x) = \begin{cases} 1 & x \geq 0 \\ -1 & x < 0 \end{cases}$

$|f| = 1$ $\lim_{x \rightarrow 0} |f| = 1 \exists$, ale $\lim_{x \rightarrow 0} f(x) \nexists$ $\lim_{x \rightarrow 0^+} f(x) \neq \lim_{x \rightarrow 0^-} f(x)$.

Pozn.: $\sin \frac{1}{x}$ nemá limitu ^{pro} $x \rightarrow 0$: Polce $x = \frac{2}{(2k+1)\pi}$

$\sin \frac{1}{x} = \sin \frac{(2k+1)\pi}{2} = -(-1)^k$. Také \exists limity vysporují

jako u Dir. fce: Vždy najdu δ (nebo k), že $(-\delta, \delta)$

$\sin \frac{1}{x}$ bude mít hodnotu + i - jedna.

4. Derivace sudé je lichá. Bud' $x_0 \in \mathbb{R}$

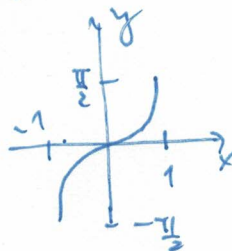
$$f'(x_0) = \lim_{h \rightarrow 0} \frac{f(x_0+h) - f(x_0)}{h} = \lim_{h \rightarrow 0} \frac{f(x_0-h) - f(x_0)}{h} =$$

$$\begin{aligned} \tilde{h} = -h & \\ = \lim_{\tilde{h} \rightarrow 0} \frac{f(x_0 + \tilde{h}) - f(x_0)}{-\tilde{h}} & \stackrel{!}{=} - \lim_{\tilde{h} \rightarrow 0} \frac{f(x_0 + \tilde{h}) - f(x_0)}{\tilde{h}} = -f'(x_0) \end{aligned}$$

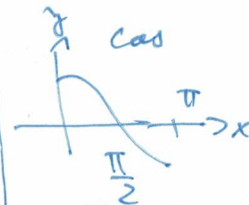
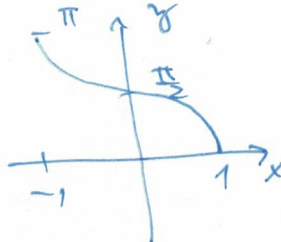
Tj. $f'(x_0) = -f'(x_0) \Rightarrow$ lichá.

Některé Elementární funkce (máme známé).

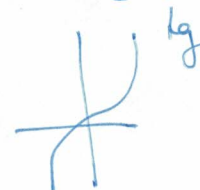
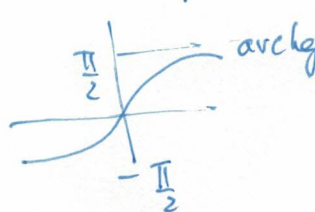
1. arcsin = sin⁻¹ $[-\frac{\pi}{2}, \frac{\pi}{2}]$



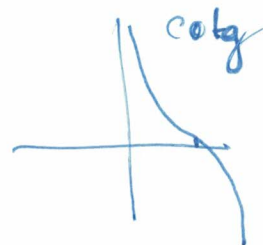
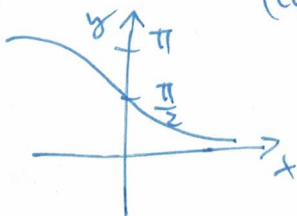
2. arccos = cos⁻¹ $[0, \pi]$



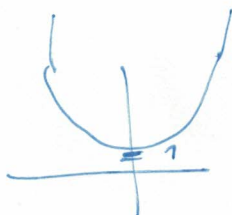
3. arctan := tan⁻¹ $[-\frac{\pi}{2}, \frac{\pi}{2}]$



4. arccot := cotan⁻¹ $[0, \pi]$

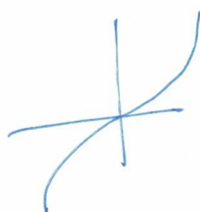


5. cosh x := $\frac{e^x + e^{-x}}{2}$ (suda')



↑ "rychlejší než parabola"

6. sinh x := $\frac{e^x - e^{-x}}{2}$ (lichá')



Derivace elementárních funkcí

2'

Viz na svých stránkách.

Vyhledejte (stáhnout si).

Poporučuji a^x i $\log_a x$ \rightarrow si umět odvodit.

Derivace:

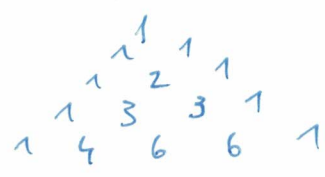
a) $(f+g)' = f' + g'$

b) $(cf)' = cf'$

c) $(fg)' = f'(x)g(x) + f(x)g'(x)$

d) $\left(\frac{f}{g}\right)'(x) = \frac{f'(x)g(x) - f(x)g'(x)}{g^2(x)}$ $\forall x, h$
 $g(x) \neq 0$

e) $(f \circ g)'(x) = f'(g(x))g'(x)$ $(f^{-1})'(x) = \frac{1}{f'(f^{-1}(x))}$



Pr: $(x^4)'(x_0) = \lim_{h \rightarrow 0} \frac{(x_0+h)^4 - x_0^4}{h} = \lim_{h \rightarrow 0} \frac{x_0^4 + 4x_0^3h + 6x_0^2h^2 + 4x_0h^3 + h^4 - x_0^4}{h}$

$$= \lim_{h \rightarrow 0} (4x_0^3 + 6x_0^2h + 4x_0h^2 + h^3) = 4x_0^3$$

$(x^n)' = nx^{n-1}$ odecne.

Pr: $\arcsin'(x) = \frac{1}{\sin'(\arcsin(x))} = \frac{1}{\cos(\arcsin(x))}$

$\frac{f}{f'} = \sin$
 $\frac{f'}{f} = \cos$

$$= \frac{1}{\sqrt{1 - \sin^2(\arcsin(x))}} = \frac{1}{\sqrt{1 - \sin(\arcsin(x)) \sin(\arcsin(x))}}$$

konvence
 $\sin^2 x \neq \sin x^2$
 u ybrz $\sin x \sin x$, tj. $(\sin x)^2$.

$$= \frac{1}{\sqrt{1-x^2}} \text{ na } (-1,1)$$

O derivaci $v \pm 1$ mahn ulnit jen \leftarrow jako o jednostranne.

1. $f(x) = \frac{2x^2}{1-x^2}$ $f'(x) = \frac{2(1-x^2) - 2x(-2x)}{(1-x^2)^2} = \frac{2x^2+2}{(1-x^2)^2}$
 $x \neq \pm 1$

2. $f(x) = \sqrt[3]{\frac{1+x}{1-x^2}} = \left(\frac{1+x}{1-x^2}\right)^{\frac{1}{3}}$ $f'(x) = \frac{1}{3} \left(\frac{1+x}{1-x^2}\right)^{-\frac{2}{3}} \cdot \left[\frac{1+x}{1-x^2}\right]' =$
 $x \neq \pm 1$ $g(x) = \frac{1+x}{1-x^2}$
 $= \frac{1}{3} \left(\frac{1+x}{1-x^2}\right)^{-\frac{2}{3}} \frac{1+2x+x^2}{(1-x^2)^2} = \frac{1}{3} \frac{(1+x)^{\frac{4}{3}}}{(1-x^2)^{\frac{4}{3}}} = \frac{1}{3} \frac{\sqrt[3]{(1+x)^4}}{(1-x)^{\frac{4}{3}}(1+x)^{\frac{4}{3}}}$
 $= \frac{1}{3} (1-x)^{-\frac{4}{3}} (1+x)$ Nebo umakem snaz: $f'(x) = -\frac{1}{3} (1-x)^{-\frac{4}{3}} \cdot (-1) = \frac{1}{3} (1-x)^{-\frac{1}{3}}$
 $f(x) = \left(\frac{1+x}{(1+x)(1-x)}\right)^{\frac{1}{3}} = (1-x)^{-\frac{1}{3}} \Rightarrow f'(x) = -\frac{1}{3} (1-x)^{-\frac{4}{3}} \cdot (-1) = \frac{1}{3} (1-x)^{-\frac{1}{3}}$

3. $f(x) = \sin(\sin(\sin x))$ $f'(x) = \cos(\sin(\sin x)) \cdot (\sin \circ \sin)'(x) =$
 $x \in \mathbb{R}$ $\begin{matrix} f & g \\ \uparrow & \uparrow \\ f & g \end{matrix}$ $\left. \begin{matrix} \text{Minodem to} \\ \text{v\u00edbec nem} \\ \text{derivace } \sin^2 x \\ \text{podobn\u00e9} \end{matrix} \right\}$
 $= \cos(\sin(\sin x)) \cos(\sin x) \cos x$
 $\begin{matrix} f'(g) & g' \end{matrix}$

4. $f(x) = 2 \lg \frac{1}{x} = e^{(\ln 2) \lg \frac{1}{x}}$ $\frac{1}{x} \neq (2k+1)\frac{\pi}{2} \quad x \neq 0$
 $f'(x) = e^{\ln 2 \lg \frac{1}{x}} (\ln 2 \lg \frac{1}{x})' = 2 \lg \frac{1}{x} \ln 2 \frac{1}{1+(\frac{1}{x})^2} \cdot \left(\frac{1}{x}\right)' =$
 $= 2 \lg \frac{1}{x} \ln 2 \frac{x^2}{1+x^2} \left(-\frac{1}{x^2}\right) \quad x \neq 0, \frac{1}{x} \neq (2k+1)\frac{\pi}{2} \quad \forall k \in \mathbb{Z}$
 $= 2 \lg \frac{1}{x} \ln 2 \frac{1}{1+x^2}$ st\u00e1le ale "dr\u00ed\u0161im" $x \neq 0$

5. $f(x) = x^{a^a} + a^{x^a} + a^{a^x}$ (bez def. oboru; tendova)

5

$f(x) = x^{(a^a)} + a^{(x^a)} + a^{(a^x)}$ "zřejná konvence.

Díval např. $(x^a)^a = x^{a \cdot a} = x^{a^2}$

$$\begin{aligned} f'(x) &= a^a x^{a^a-1} + (e^{\ln a \cdot x^a})' + (e^{a^x \ln a})' = \\ &= a^a x^{a^a-1} + a^{x^a} \ln a \cdot a x^{a-1} + a^{a^x} (a^x \ln a)' = \\ &= a^a x^{a^a-1} + a^{x^a} \ln a \cdot a \cdot x^{a-1} + a^{a^x} \ln a (e^{\ln a x})' = \\ &= a^a x^{a^a-1} + a^{x^a} a \ln a x^{a-1} + a^{a^x} \ln a^2 a^x. \end{aligned}$$

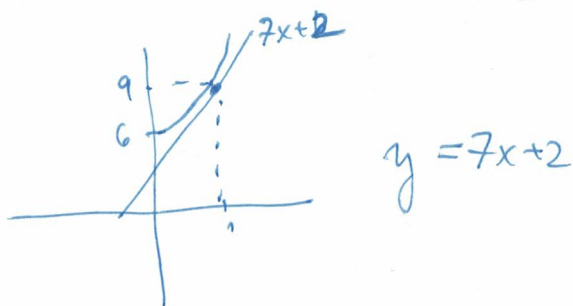
Takže pozor: $(a^{x^a})' \neq x^a a^{x^a-1}$.

6. Spočítejte rovnici tečny k $f(x) = x^3 + 2x^2 + 6$ v $x = 1$

$f'(x) = 3x^2 + 4x$, $f'(1) = 7$

$y = 7x + b$

$y(1) = 7 + b = f(1) = 9 \Rightarrow b = 2$



! Dělení polynomů se zbytkem: $S(x) = Q(x)P(x) + R(x)$
 $\forall S \text{ a } Q \neq 0 \exists P \exists R \text{ deg } R < \text{deg } Q$, aby

Prk.:
$$\begin{array}{r} x^4 + 2x^2 + x + 2 : (x^2 + 1) = x^2 + 1 \dots = S \\ - (x^4 + x^2) \\ \hline x^2 + x + 2 \\ - (x^2 + 1) \\ \hline x + 1 \end{array}$$

Tj.
$$\frac{x^4 + 2x^2 + x + 2}{x^2 + 1} = x^2 + 1 + \frac{x+1}{x^2+1}$$

 where $P(x) = x^2 + 1$ and $Q(x) = x^2 + 1$.

$$x^5 + 3x + 6 : (x^2 + 1) = x^3 - x^3 + x^2 - x + 4 + \frac{2}{x+1}$$

$$\frac{-(x^5 + x^4)}{}$$

$$-x^4 + 3x + 6$$

$$\frac{-(-x^4 - x^3)}{}$$

$$x^3 + 3x + 6$$

$$\frac{-(x^3 + x^2)}{}$$

$$-x^2 + 3x + 6$$

$$\frac{-(-x^2 - x)}{}$$

$$4x + 6$$

$$\frac{-(4x + 4)}{}$$

$$+2$$

$\deg 2 < \deg (x+1)$

Primitivní funkce

Pro f hledáme F , aby $F' = f$. Funkce' jidnomacua': $(F+c)' = f$

Pr.: Pro $f(x) = 2x$ hledáme F , aby $F' = 2x$.

Víme $(x^2)' = 2x$. Zkusme $F(x) = \frac{1}{2}x^2$. $F'(x) =$

$$= \frac{1}{2} \cdot 2x = x.$$

$$\text{Řešme } \int x dx = \frac{1}{2}x^2 + C$$

Pr.: $f(x) = \frac{1}{2} \sin x$ | Nejdrův zkusme $\cos x$; $(\cos x)' = -\sin x$ | Tak $(-\cos x)' = -1 \cdot (-\sin x) = \sin x$

Jestli přidáme $\frac{1}{2} \rightsquigarrow F(x) = \frac{-\cos x}{2}$

$$F'(x) = -\frac{1}{2} \cdot (-\sin x) = \frac{1}{2} \sin x. \quad \text{Přidáme tedy}$$

$$\int \frac{1}{2} \sin x dx = -\frac{\cos x}{2} + C.$$