$$\mathcal{F}(e^{-\pi x^2})(\xi) = \int_{\mathbb{R}} e^{-\pi x^2} e^{-2\pi i x\xi}\,dx = \int_{\mathbb{R}} e^{-\pi x^2 - 2\pi i x\xi} = ?$$

$\xi > 0$ Cauchy

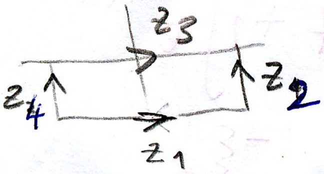
$$0 = \oint_{C_R} e^{-\pi z^2 - 2\pi i z\xi}\,dz = \oint_{C_R} e^{-\pi(z^2+2\pi i z\xi - \xi^2)} e^{-\pi\xi^2}\,dz = \oint_{C_R} e^{-\pi(z+i\xi)^2} e^{-\pi\xi^2}\,dz$$

$C_R: z_1 = t - i\xi,\ t\in[-R,R]$

$z_2 = R+it,\ t\in[-\xi,0]$

$z_3 = t,\ t\in[-R,R]$

$z_4 = -R+it,\ t\in[-\xi,0]$



$$= e^{-\pi\xi^2}\oint_{C_R} e^{-\pi(z+i\xi)^2}\,dz$$

$$= e^{-\pi\xi^2}\Big[\underbrace{+\int_{-R}^{R} e^{-\pi t^2}\,dt}_{I_1} + \underbrace{i\int_{-\xi}^{0} e^{-\pi(R+it+i\xi)^2}\,dt}_{I_2} +$$

$$\underbrace{-\int_{-R}^{R} e^{-\pi(t+i\xi)^2}\,dt}_{I_3} \underbrace{- i\int_{-\xi}^{0} e^{-\pi(-R+it+i\xi)^2}\,dt}_{I_4}\Big]$$

$R\to\infty$:

$$\lim_{R\to\infty} I_1 = e^{-\pi\xi^2}\int_{-\infty}^{+\infty} e^{-\pi t^2}\,dt \to \text{Laplaceův integrál} = e^{-\pi\xi^2}\ \Big|\ \underline{\text{subst}}\ \underline{\text{Fubini}}\ \Big|$$

$$\lim_{R\to\infty} I_2 = \lim_{R\to\infty}\int_{-\xi}^{0} e^{-\pi R^2} e^{2Ri(t+\xi)} e^{-(t+\xi)^2}\,dt = \int_{-\xi}^{0} 0 = 0$$

PP : majoranta $\left|e^{-\pi R^2} e^{2Ri(t+\xi)} e^{-(t+\xi)^2}\right| =$

$= \left|e^{-\pi R^2} e^{-(t+\xi)^2}\right| \leq e^{-\pi R^2}$ integrab.

$$\lim_{R\to\infty} I_3 = \int_{-\infty}^{+\infty} e^{-\pi t^2 - 2\pi i\xi t + \pi\xi^2}\,dt = e^{\pi\xi^2}\mathcal{F}(e^{-\pi x^2})(\xi)$$

$$\lim_{R\to\infty} I_4 = \lim_{R\to\infty}\int_{-\xi}^{0} e^{-\pi(-R+it+i\xi)^2}\,dt = \int_{-\xi}^{0} 0\,dt = 0$$

PP: majoranta $e^{-\pi R^2} e^{\pi\xi^2}$

$$0 = \overset{+}{-} e^{-\pi\xi^2} \overset{-}{+} e^{-\pi\xi^2} e^{\pi\xi^2}\mathcal{F}(e^{-\pi x^2})(\xi)$$

$$\boxed{e^{-\pi\xi^2} = \mathcal{F}(e^{-\pi x^2})(\xi)}$$

$h_0(x) = e^{-\pi x^2}$ $\quad \mathcal{F}(h_0) = h_0$ $\quad\Big|$ h_n

$\mathcal{F}(h_n) = h_n i^n$ (Wiener)

$\mathcal{F}^4 = Id$