$$\lim_{n\to\infty} \sqrt[n]{\sqrt{3^n+2\cdot 2^n} - \sqrt{3^n+2^n}} = \lim_{n\to\infty} \frac{\sqrt{3\cdot 3^n+4\cdot 2^n} - \sqrt{3\cdot 3^n+2\cdot 2^n}}{\sqrt{3^n+2\cdot 2^n} - \sqrt{3^n+2^n}}$$

I

$$\sqrt{3^n+2\cdot 2^n} > \sqrt{3^n+2^n}$$

$$\Rightarrow a_n > 0 : \lim_{n\to\infty} \frac{a_{n+1}}{a_n} = A \not\Leftrightarrow \lim_{n\to\infty} \sqrt[n]{a_n} = A$$

$$\Rightarrow$$

zlomek rozšíříme podle vzorce $(A-B)(A+B) = A^2 - B^2$

$$= \lim_{n\to\infty} \frac{\left(3\cdot 3^n + 4\cdot 2^n - 3\cdot 3^n - 2\cdot 2^n\right)\left(\sqrt{3^n+2\cdot 2^n} + \sqrt{3^n+2^n}\right)}{\left(3^n+2\cdot 2^n - 3^n - 2^n\right)\left(\sqrt{3\cdot 3^n+4\cdot 2^n} + \sqrt{3\cdot 3^n+2\cdot 2^n}\right)} =$$

$$= \lim_{n\to\infty} \frac{2\cdot \cancel{2^n}}{\cancel{2^n}} \cdot \frac{\left(\sqrt{3^n+2\cdot 2^n} + \sqrt{3^n+2^n}\right)}{\left(\sqrt{3\cdot 3^n+4\cdot 2^n} + \sqrt{3\cdot 3^n+2\cdot 2^n}\right)} \overset{\text{VOAL}}{=} \lim_{n\to\infty} 2 \cdot \lim_{n\to\infty} \frac{\sqrt{3^n+2\cdot 2^n} + \sqrt{3^n+2^n}}{\sqrt{3\cdot 3^n+4\cdot 2^n} + \sqrt{3\cdot 3^n+2\cdot 2^n}}$$

$$= 2 \cdot \lim_{n\to\infty} \frac{\sqrt{3^n}\cdot\sqrt{1+2\cdot\left(\frac{2}{3}\right)^n} + \sqrt{3^n}\cdot\sqrt{1+\left(\frac{2}{3}\right)^n}}{\sqrt{3}\cdot\sqrt{3^n}\cdot\sqrt{1+\frac{4}{3}\cdot\left(\frac{2}{3}\right)^n} + \sqrt{3}\cdot\sqrt{3^n}\cdot\sqrt{1+\frac{2}{3}\cdot\left(\frac{2}{3}\right)^n}} =$$

$$= 2 \cdot \lim_{n\to\infty} \frac{\cancel{\sqrt{3^n}}\left(\sqrt{1+2\cdot\left(\frac{2}{3}\right)^n} + \sqrt{1+\left(\frac{2}{3}\right)^n}\right)}{\sqrt{3}\cdot\cancel{\sqrt{3^n}}\left(\sqrt{1+\frac{4}{3}\cdot\left(\frac{2}{3}\right)^n} + \sqrt{1+\frac{2}{3}\cdot\left(\frac{2}{3}\right)^n}\right)} =$$

$$\overset{\text{VOAL}}{=} 2 \cdot \lim_{n\to\infty} \frac{1}{\sqrt{3}} \cdot \lim_{n\to\infty} \frac{\sqrt{1+2\cdot\left(\frac{2}{3}\right)^n} + \sqrt{1+\left(\frac{2}{3}\right)^n}}{\sqrt{1+\frac{4}{3}\cdot\left(\frac{2}{3}\right)^n} + \sqrt{1+\frac{2}{3}\left(\frac{2}{3}\right)^n}} \overset{\text{VOAL}}{=}$$

$$\overset{\text{VOAL}}{=} 2 \cdot \frac{1}{\sqrt{3}} \cdot \frac{\sqrt{\lim 1 + \lim 2\cdot \lim\left(\frac{2}{3}\right)^n} + \sqrt{\lim 1 + \lim\left(\frac{2}{3}\right)^n}}{\sqrt{\lim 1 + \lim\frac{4}{3}\cdot\lim\left(\frac{2}{3}\right)^n} + \sqrt{\lim 1 + \lim\frac{3}{3}\cdot\lim\left(\frac{2}{3}\right)^n}} =$$

$$\lim a_n^q = \left(\lim a_n\right)^q \quad q \in \mathbb{Q},\ a_n > 0$$

$$\lim_{n\to\infty} \sqrt{\underbrace{1+2\cdot\left(\tfrac{2}{3}\right)^n}_{1}} > 0$$

$\Rightarrow$ vlastní limita

$$\overset{\text{102P}}{=} 2 \cdot \frac{1}{\sqrt{3}} \cdot \frac{\sqrt{1} + 2\cdot 0 + \sqrt{1} + 0}{\sqrt{1} + \frac{4}{3}\cdot 0 + \sqrt{1} + \frac{2}{3}\cdot 0} = 2 \cdot \frac{1}{\sqrt{3}} \cdot \frac{1+1}{1+1} = \underline{\underline{\frac{2}{\sqrt{3}}}}$$

$$\lim \left(\frac{2}{3}\right)^n = 0$$

$\rightarrow$ netřeba ověřovat

$$0 \le \left(\frac{2}{3}\right)^n = \frac{1}{\left(\frac{3}{2}\right)^n} \le \underbrace{\frac{1}{n}}_{\to 0}$$