Limita posloupnosti - komplexní úloha V

Určete limitu posloupnosti

$$\lim_{n\to\infty} \sqrt[n]{n^2 + n^3 + n^4 + 2^n + 3^n + 4^n}.$$

Řešení

$$\lim_{n\to\infty} \sqrt[n]{n^2+n^3+n^4+2^n+3^n+4^n} = \lim \frac{(n+1)^2+(n+1)^3+(n+1)^4+2^{n+1}+3^{n+1}+4^{n+1}}{n^2+n^3+n^4+2^n+3^n+4^n} =$$

$$= \lim_{n\to\infty} \frac{4^n\cdot\left(4+\frac{(n+1)^2}{4^n}+\frac{(n+1)^3}{4^n}+\frac{(n+1)^4}{4^n}+\frac{2^{n+1}}{4^n}+\frac{3^{n+1}}{4^n}\right)}{4^n\cdot\left(1+\frac{n^2}{4^n}+\frac{n^3}{4^n}+\frac{n^4}{4^n}+\frac{2^n}{4^n}+\frac{3^n}{4^n}\right)} =$$

$$\overset{\text{VOAL}}{=} \frac{\lim 4+\lim\left(\frac{(n+1)^2}{4^n}+\frac{(n+1)^3}{4^n}+\frac{(n+1)^4}{4^n}+2\cdot\frac{1}{2^n}+3\left(\frac{3}{4}\right)^n\right)}{\lim 1+\lim\left(\frac{n^2}{4^n}+\frac{n^3}{4^n}+\frac{n^4}{4^n}+\frac{1}{2^n}+\left(\frac{3}{4}\right)^n\right)} \overset{\text{VOAL}}{=} \frac{4+0+0+0+2\cdot 0+3\cdot 0}{1+0+0+0+0+0}$$

$$= \underline{\underline{4}}$$