Limita posloupnosti - komplexní úloha X

Určete limitu posloupnosti

$$\lim_{n\to\infty} \sqrt[n]{\frac{4^n + 3^n \sin(2^n)}{5^n + 4^n \cos(n!)}}.$$

Řešení

$$\lim_{n\to\infty} \sqrt[n]{\frac{4^n + 3^n \sin(2^n)}{5^n + 4^n \cos(n!)}} \overset{\text{VOAL}}{=} \lim_{n\to\infty} \sqrt[n]{\frac{4^n}{5^n}} \cdot \lim_{n\to\infty} \sqrt[n]{\frac{1+\left(\frac{3}{4}\right)^n \sin(2^n)}{1+\left(\frac{4}{5}\right)^n \cos(n!)}} =$$

$$\overset{\text{VOAL}}{=} \cancel{\lim_{n\to\infty} \frac{4}{5} \cdot \lim_{n\to\infty} \sqrt[n]{\frac{1+0}{1+0}}} = \frac{4}{5} \cdot 1 = \frac{4}{5}$$

$$-1 \leq \cancel{\lim_{n\to\infty}} \sin 2^n \leq 1 \qquad \text{VO2P}$$

$$\lim_{n\to\infty} \left(\frac{3}{4}\right)^n \cdot (-1) \leq \lim_{n\to\infty} \left(\frac{3}{4}\right)^n \sin 2^n \leq \lim_{n\to\infty} 1 \left(\frac{3}{4}\right)^n$$

$$0 \leq \lim_{n\to\infty} \left(\frac{3}{4}\right)^n \sin 2^n \leq 0$$

$$\lim \left(\frac{3}{4}\right)^n \sin 2^n = 0$$

$$-1 \leq \lim_{n\to\infty} \cos(n!) \leq 1$$

$$\lim_{n\to\infty} \left(\frac{4}{5}\right)^n \cdot (-1) \leq \lim_{n\to\infty} \left(\frac{4}{5}\right)^n \cos(n!) \leq \lim_{n\to\infty} \left(\frac{4}{5}\right)^n$$

$$0 \leq \lim_{n\to\infty} \left(\frac{4}{5}\right)^n \cos(n!) \leq 0$$

$$\lim_{n\to\infty} \left(\frac{4}{5}\right)^n \cos(n!) = 0$$

$$1 + (-1) \cdot \left(\frac{3}{4}\right)^n \leq \left(\frac{3}{4}\right)^n \sin 2^n + 1 \leq 1 + \left(\frac{3}{4}\right)^n$$

$$\leq 1 + \frac{3}{4}$$

$$1 - \frac{3}{4} \leq$$

$$\sqrt[n]{1-\frac{3}{4}} \to 1 \qquad \leq \sqrt[n]{\left(\frac{3}{4}\right)^n \sin 2^n + 1} \leq \qquad \sqrt[n]{1+\frac{3}{4}} \to 1$$