HZP $\lim\limits_{n\to\infty} 1+\left(\frac{3}{4}\right)^n \sin 2^n = 1$

$$\lim_{n\to\infty} 1+\left(\frac{4}{5}\right)^n \cos(n!) = 1$$

$$1+\left(-\frac{4}{5}\right) \leq 1+\left(\frac{4}{5}\right)\cos(n!) \leq 1+\frac{4}{5}$$

$$\sqrt[n]{1+\left(-\frac{4}{5}\right)} \leq \sqrt[n]{\left(\frac{4}{5}\right)^n\cos(n!)+1} \leq \sqrt[n]{1+\frac{4}{5}}$$

$$\sqrt[n]{\frac{1}{5}} \leq \sqrt[n]{\left(\frac{4}{5}\right)\cos(n!)+1} \leq \sqrt[n]{\frac{9}{5}} \to 1$$

$$\to 1$$

$$\lim_{n\to\infty} \sqrt[n]{\frac{4^n}{5^n}} \cdot \lim_{n\to\infty} \sqrt[n]{\frac{1+\left(\frac{3}{4}\right)^n \sin(2^n)}{1+\left(\frac{4}{5}\right)^n\cos(n!)}} =$$

$$\overset{\text{VOAL}}{=} \frac{4}{5} \cdot \cancel{\lim_{n\to\infty} \sqrt[n]{\frac{1+0}{1+0}}} = \frac{4}{5}\cdot\frac{1}{1} = \underline{\underline{\frac{4}{5}}}$$

$$\frac{\lim\limits_{n\to\infty} \sqrt[n]{1+\left(\frac{3}{4}\right)^n \sin(2^n)}}{\lim\limits_{n\to\infty} \sqrt[n]{1+\left(\frac{4}{5}\right)^n \cos(n!)}}$$