$\frac{10.04}{\text{OK}}$ $$\int_{-\infty}^{0} \frac{x}{x^3-1}\,dx =$$

$$= \int_{-\infty}^{0} \frac{1}{3}\left( \frac{1}{x-1} - \frac{1}{2}\frac{\overset{+1-3}{2x-2}}{x^2+x+1} \right) dx$$

$$x = A(x^2+x+1) + (Bx+C)(x-1)$$

$x=1 \Rightarrow A = 1/3 \Rightarrow B = -1/3$, $C = 1/3$ abs. čl.

$$= \frac{1}{3}\left[ \ln \frac{|x-1|}{\sqrt{x^2+x+1}} \right]_{-\infty}^{0} + \left[ \frac{1}{2}\cdot\frac{2}{\sqrt{3}} \operatorname{arctg} \frac{2x+1}{\sqrt{3}} \right]_{-\infty}^{0}$$

$$= 0 + \frac{1}{\sqrt{3}}\left( \frac{\pi}{2} + \frac{\pi}{6} \right) = \underline{\underline{\frac{2\pi}{9}\sqrt{3}}}$$

$\frac{10.03}{\text{OK}}$ $$\int_{4}^{+\infty} \frac{x}{(x-1)(x-2)(x-3)}\,dx =$$

$$= \int_{4}^{\infty} \frac{1}{2}\left( \frac{1}{x-1} + \frac{-4}{x-2} + \frac{3}{x-3} \right)$$

$$x = A(x-2)(x-3) + B(x-1)(x-3) + C(x-1)(x-2)$$

$x=1 \Rightarrow A = \frac{1}{2}$

$x=2 \Rightarrow B = -2$

$x=3 \Rightarrow C = 3/2$

$$= \frac{1}{2}\left[ \ln \frac{(x-1)(x-3)^3}{(x-2)^4} \right]_{4}^{\infty} = -\frac{1}{2} \ln \frac{3 \cdot 1^3}{2^4} =$$

$$= \underline{\underline{\frac{1}{2} \ln \frac{16}{3}}}$$

10.25. ★ $\int_0^{\pi} \frac{\sin x}{\cos^2 x + 1}\,dx =$

$t = \cos x,$

$= \int_{-1}^{1} \frac{1}{t^2+1}\,dt = \left[\operatorname{arctg} t\right]_{-1}^{1} = \frac{\pi}{2}.$

10.24. ★ $\int_0^{+\infty} \frac{e^{-1/x}}{x^2}\,dx \underset{t=1/x}{=} \int_0^{+\infty} e^{-t} = \left[-e^{-t}\right]_0^{\infty}$

$= 1.$

10.23. $\int_1^{\sqrt{3}} \frac{dx}{(x^2+1)\operatorname{arctg} x} \underset{t=\operatorname{arctg} x}{=} \int_{\pi/4}^{\pi/3} \frac{dt}{t} = \ln\frac{4}{3}.$

10.22. $\int_{-\infty}^{0} x e^{-x^2}\,dx \underset{t=x^2}{=} \int_{\infty}^{0} \frac{1}{2} e^{-t}\,dt =$

$= \left[-\frac{1}{2}e^{-t}\right]_{\infty}^{0} = -\frac{1}{2}.$

10.21.

**10.30.** OK

$$\int_0^9 \frac{dx}{\sqrt{x+16}-\sqrt{x}} = \int_0^9 \frac{\sqrt{x+16}+\sqrt{x}}{16}\,dx$$

$$= \frac{1}{16}\left[\frac{(x+16)^{3/2}}{3/2} + \frac{x^{3/2}}{3/2}\right]_0^9 =$$

$$= \frac{1}{16}\cdot\frac{2}{3}\left[5^3 + 3^3 - 4^3\right] = \frac{125+27-64}{24} = \frac{88}{24} = \frac{11}{3}.$$

**10.27.** ?

$$\int_{-\infty}^{+\infty} \frac{e^x}{e^{2x}+e^x+1}\,dx = \qquad t=e^x$$

$$= \int_0^{+\infty} \frac{dt}{t^2+t+1} = \left[\frac{2}{\sqrt{3}}\operatorname{arctg}\frac{2t+1}{\sqrt{3}}\right]_0^{\infty} =$$

$$= \frac{2}{\sqrt{3}}\left(\frac{\pi}{2} - \frac{\pi}{6}\right) = \frac{2\sqrt{3}}{9}\pi$$

**10.28.**

$$\int_0^{+\infty} \frac{dx}{\sqrt{e^{2x}+1}} =$$

$t=e^{-x}$, $dt=-e^{-x}dx$

$$= \int_0^1 \frac{dt}{\sqrt{1+t^2}} = \left[\ln\left(t+\sqrt{t^2+1}\right)\right]_0^1 = \ln(1+\sqrt{2})$$

**10.29**

$$\int_{-\infty}^{+\infty} \frac{e^x}{e^{2x}+3e^x+3}\,dx =$$

$t=e^x$

$$= \int_0^{+\infty} \frac{dt}{t^2-3t+3} = \int_0^{+\infty} \frac{dt}{\left(t-\frac{3}{2}\right)^2+\frac{3}{4}} =$$

$$= \left[\frac{2}{\sqrt{3}}\operatorname{arctg}\frac{2t-3}{\sqrt{3}}\right]_0^{+\infty} =$$

$$= \frac{2}{\sqrt{3}}\left(\frac{\pi}{2} + \frac{\pi}{3}\right) = \frac{5\pi}{3\sqrt{3}} = \frac{5\sqrt{3}}{9}\pi$$

**10.26.**

$$\int_{-\infty}^{+\infty} \frac{e^x}{e^{2x}+e^{-2x}}\,dx = \cancel{\int_{-\infty}^{+\infty}\frac{e^{-x}\,dx}{1+e^{-4x}}}$$

$$\cancel{= \int_0^{+\infty}\frac{dt}{t^2+\frac{1}{t^2}}} = \int_0^{+\infty}\frac{t^2}{t^4+1}\,dt =$$

$$\cancel{= \int_0^1 \frac{dt}{1+t^4}} =$$

$t=e^x$

$$= \int_0^{+\infty} \frac{t^2\,dt}{t^4+1}$$

and as 10.16.

**10.35.** $\int_0^{\pi} \sin^2 x \cos^2 x \, dx =$

*

$\int_0^{\pi} \frac{1}{4} \sin^2 2x \, dx =$

$t = 2x$

$= \int_0^{2\pi} \frac{1}{8} \sin^2 \cancel{2t}\, t \, dt = \frac{1}{8} \left[ \frac{t + \sin t \cos t}{2} \right]_0^{2\pi}$

$= \frac{\pi}{8}.$

---

**10.34.** $\int_{-\pi/2}^{\pi/2} x^3 \sin x \, dx \overset{PP}{=}$

*

$\overset{PP}{=} \left[ -x^3 \cos x \right]_{-\pi/2}^{\pi/2} + \int_{-\pi/2}^{\pi/2} 3x^2 \cos x \, dx \overset{PP}{=}$

$\underset{0}{\overset{\shortparallel}{}}$

$= \left[ 3x^2 \sin x \right]_{-\pi/2}^{\pi/2} - \int_{-\pi/2}^{\pi/2} 6x \sin x \, dx =$

$\overset{=}{6} \frac{3}{2}\pi^2$

$= \frac{3}{2}\pi^2 + \boxed{0} - 6 \int_{-\pi/2}^{\pi/2} \cos x \, dx = \frac{3}{2}\pi^2 - 12$

---

**10.33.** $\int_0^{\pi} \sin^3 x \, dx = \int_0^{\pi} (1 - \cos^2 x) \sin x \, dx$

*

$t = \cancel{\sin x}\ \cos x$

$= \int_{-1}^{1} (1 - u^2) \, du = \left[ u - \frac{u^3}{3} \right]_{-1}^{1} =$

$= \frac{2}{3} + \frac{2}{3} = \frac{4}{3}.$

---

**10.~~33~~32** $\int_0^{\pi/4} \operatorname{tg} x \, dx = \int_0^{\pi/4} \frac{\sin x}{\cos x} \, dx$

*

$t = \cos x$

$= \int_{\sqrt{2}/2}^{1} \frac{dt}{t} = \left[ \ln t \right]_{\sqrt{2}/2}^{1} = - \ln \frac{1}{\sqrt{2}} = \ln \sqrt{2}$

$= \frac{1}{2} \ln 2.$

---

**10.31.** $\int_1^{+\infty} \frac{dx}{x(\ln^2 x + 2\ln x + 2)^2} =$

$t = \ln x$

$= \int_0^{\infty} \frac{dt}{(t^2 + 2t + 2)^2} = \int_0^{\infty} \frac{dt}{((t+1)^2 + 1)^2} =$

$t + 1 = \operatorname{tg} u$

$= \int_{\pi/4}^{\pi/2} \cos^2 u \, du = \left[ \frac{u + \cos u \sin u}{2} \right]_{\pi/4}^{\pi/2} = \frac{\pi}{8} - \frac{1}{4}.$

**10.42.** $\int_{-1}^{1} x^2 e^{-x}\,dx \overset{PP}{=}$

$*$

$= \left[-x^2 e^{-x}\right]_{-1}^{1} + \int_{-1}^{1} 2x e^{-x}\,dx \overset{PP}{=}$

$= (e - e^{-1}) + \left[-2x e^{-x}\right]_{-1}^{1} + 2\int_{-1}^{1} e^{-x}\,dx =$

$= (e - e^{-1}) + (2e - 2e^{-1}) + 2\left[-e^{-x}\right]_{-1}^{1} =$

$-2e$

$= (e - 5e^{-1})$

---

**10.41.** $\int_0^{+\infty} \left(3x\cos x^3 - \frac{1}{x^2}\sin x^3\right) dx$

$*$

$\int_0^{+\infty} \frac{1}{x^2}\sin x^3 \overset{PP}{=} \left[-\frac{1}{x}\sin x^3\right]_0^{\infty} + \int_0^{+\infty} \frac{1}{x} 3x^2 \cos x^3\,dx$

$= 0$

$= 0 + \int_0^{\infty} 3x\cos x^3\,dx$

$\Rightarrow \underline{I = 0.}$ $\left[\text{Je nutné ukázat, že existuje!! DIRICHLET!!}\right]$

---

**10.40.** $\int_0^{\pi/4} \sqrt{\cos x - \cos^3 x}\,dx =$

$= \int_0^{\pi/4} \sqrt{\cos x}\,\sin x\,dx =$

$t = \cos x$

$= \int_{\sqrt{2}/2}^{1} \sqrt{t}\,dt = \left[\frac{t^{3/2}}{3/2}\right]_{\sqrt{2}/2}^{1} =$

$= \underline{\tfrac{2}{3}\left(1 - \sqrt{\tfrac{\sqrt{8}}{8}}\right)}$

---

**10.39.** $\int_{1/3\pi}^{1/\pi} \frac{1}{x^2}\sin^2\frac{1}{x}\,dx$

$*$ D

$t = \frac{1}{x}, \quad dt = -\frac{1}{x^2}dx$

$= \int_{\pi}^{3\pi} \sin^2 t\,dt = 2\cdot\int_{\pi}^{2\pi} \sin^2 t\,dt =$

LEPENÍ!!

$= 2\cdot\left[\frac{t + \sin t\cos t}{2}\right]_{\pi}^{2\pi}$

NENÍ NUTNÉ!!

$= \underline{\pi.}$

je ZPF = PF spoj.

**10.49.** $\int_0^1 \arccos^2 x \, dx$

$\cancel{t} \quad x = \cos t, \quad dx = -\sin t \, dt$

$$\int_0^{\pi/2} t^2 \sin t \, dt \overset{PP}{=} \Big[\underbrace{t^2 \cos t}_{=0}\Big]_0^{\pi/2} - \int_0^{\pi/2} 2t\cos t \, dt =$$

$$= -\Big[\underbrace{-2t \sin t}_{=\pi}\Big]_0^{\pi/2} - \int_0^{\pi/2} 2\sin t \, dt =$$

$$= \pi - 2\big[-\cos t\big]_0^{\pi/2} = \underline{\underline{\pi - 2.}}$$

---

**10.48.** $\int_0^1 x \arcsin x \, dx =$

OK

$x = \sin t, \quad dx = \cos t \, dt$

$$= \int_0^{\pi/2} t \sin t \cos t \, dt =$$

$$= \frac{1}{4}\int_0^{\pi/2} 2t \sin 2t \, dt = \frac{1}{8}\int_0^{\pi} u \sin u \, du =$$

$u = 2t, \quad du = 2\,dt$

$$\overset{PP}{=} \frac{1}{8}\Big[\underbrace{-u\cos u}_{=+\pi}\Big]_0^{\pi} + \frac{1}{8}\int_0^{\pi} \cos u \, du = +\frac{\pi}{8} + \frac{1}{8}\big[\sin u\big]_0^{\pi} =$$

$$= \underline{+\frac{\pi}{8}}$$

---

**10.47.** $\int_1^{+\infty} \frac{1 - \ln x}{x^2} \, dx$

OK

$t = \ln x, \quad dt/dx = 1/x, \quad x = e^t$

$$= \int_0^{+\infty} \frac{1-t}{e^t} \, dt = \int_0^{+\infty} \left(e^{-t} - te^{-t}\right) dt =$$

$$= \Big[\underbrace{-e^{-t}}_{=1}\Big]_0^{\infty} - \int_0^{+\infty} te^{-t} \, dt \overset{PP}{=}$$

$$= 1 - \Big[\underbrace{-te^{-t}}_{=0}\Big]_0^{+\infty} - \int_0^{+\infty} e^{-t} \, dt = \underline{\underline{\cancel{2}0}}$$

---

**10.46.** $\int_1^e \frac{1 + \ln x}{x} \, dx$

*

$t = \ln x, \quad dt = \frac{1}{x} dx$

$$= \int_0^1 (1+t)\, dt = \Big[t + \frac{t^2}{2}\Big]_0^1 = \underline{\underline{3/2.}}$$

**10.45.** $\displaystyle\int_0^1 x^2 \ln(1+x^2)\,dx \overset{PP}{=}$

*

$$= \left[\frac{x^3}{3}\ln(1+x^2)\right]_0^1 - \int_0^1 \frac{2x^4}{3(1+x^2)}\,dx =$$

$= \frac{1}{3}\ln 2$

$$= \tfrac{1}{3}\ln 2 - \tfrac{2}{3}\int_0^1 \left(\frac{x^4+x^2}{1+x^2} - \frac{x^2+1}{x^2+1} + \frac{1}{x^2+1}\right)dx$$

$$= \tfrac{1}{3}\ln 2 - \tfrac{2}{3}\int_0^1 \left(x^2 - 1 + \frac{1}{x^2+1}\right)dx =$$

$$= \tfrac{1}{3}\ln 2 - \tfrac{2}{3}\left[\frac{x^3}{3} - x + \operatorname{arctg} x\right]_0^1 =$$

$$= \tfrac{1}{3}\ln 2 - \tfrac{2}{3}\left[-\tfrac{2}{3} + \tfrac{\pi}{4}\right] =$$

$$= \underline{\underline{\tfrac{1}{3}\ln 2 + \tfrac{4}{9} - \tfrac{\pi}{6}}}.$$

**10.44.** $\displaystyle\int_{-\pi}^{\pi/2} \frac{\cos^3 x}{\sqrt[3]{\sin x}}\,dx = \int_{-\pi}^{-\pi/2} + \int_{-\pi/2}^{\pi/2}$

*

LEPEN!  $t = \sin x, \quad dt = \cos x\,dx$

$$= \int_0^{-1} \dots + \int_{-1}^{1} \dots =$$

$$= \int_0^1 \frac{1-t^2}{\sqrt[3]{t}}\,dt = \left[\frac{t^{2/3}}{2/3} - \frac{t^{8/3}}{8/3}\right]_0^1 =$$

$$= \frac{3}{2} - \frac{3}{8} = \underline{\underline{\frac{9}{8}}}.$$

---

**10.43.** $\displaystyle\int_0^{+\infty} \frac{dx}{\sqrt{e^x-1}} =$

*

$t = \sqrt{e^x-1}, \quad dt = \dfrac{1\cdot e^x}{2\sqrt{e^x-1}}\,dx$

$t^2+1 = e^x$

$$= \int_0^{+\infty} \frac{2}{t^2+1}\,dt = 2\left[\operatorname{arctg} t\right]_0^{\infty} = \underline{\underline{\pi}}.$$

10.49. $\int_0^1 \arccos^2 x \, dx$

$\cancel{t} \; x = \cos t ; \quad dx = -\sin t \, dt$

$$\int_0^{\pi/2} t^2 \sin t \, dt \overset{PP}{=} \Big[\underbrace{t^2 \cos t}_{=0}\Big]_0^{\pi/2} - \int_0^{\pi/2} 2t \cos t \, dt =$$

$$= -\Big[\underbrace{-2t \sin t}_{=\pi}\Big]_0^{\pi/2} - \int_0^{\pi/2} 2 \sin t \, dt =$$

$$= \pi - 2\left[-\cos t\right]_0^{\pi/2} = \underline{\underline{\pi - 2}}.$$

10.48. $\int_0^1 x \arcsin x \, dx =$

OK

$\cancel{x} = \sin t, \quad dx = \cos t \, dt$

$$= \int_0^{\pi/2} t \sin t \cos t \, dt =$$

$$= \frac{1}{4}\int_0^{\pi/2} 2t \sin 2t \, dt = \frac{1}{8}\int_0^{\pi} u \sin u \, du =$$

$u = 2t, \; du = 2\,dt$

$$\overset{PP}{=} \frac{1}{8}\Big[\underbrace{u \cos u}_{=+\pi}\Big]_0^{\pi} + \frac{1}{8}\int_0^{\pi} \cos u \, du = +\frac{\pi}{8} + \frac{1}{8}\left[\sin u\right]_0^{\pi} = \underline{\underline{+\frac{\pi}{8}}}$$

10.47. $\int_1^{+\infty} \frac{1 - \ln x}{x^2} \, dx$

OK

$t = \ln x, \quad dt/dx = 1/x, \quad x = e^t$

$$= \int_0^{+\infty} \frac{1-t}{e^t} \, dt = \int_0^{+\infty} \left(e^{-t} - t e^{-t}\right) dt =$$

$$= \Big[\underbrace{-e^{-t}}_{=1}\Big]_0^{\infty} - \int_0^{+\infty} t e^{-t} \, dt \overset{PP}{=}$$

$$= 1 - \Big[\underbrace{-t e^{-t}}_{=0}\Big]_0^{+\infty} - \int_0^{+\infty} e^{-t} \, dt = \underline{\underline{\cancel{2}0}}$$

10.46. $\int_1^e \frac{1 + \ln x}{x} \, dx$

*

$t = \ln x, \quad dt = \frac{1}{x} dx$

$$= \int_0^1 (1+t) \, dt = \left[t + \frac{t^2}{2}\right]_0^1 = \underline{\underline{3/2}}.$$

**10.64.** *

$$\int_0^1 x^5\sqrt{1-x^2}\,dx =$$

$t = 1-x^2, \quad dt = -2x\,dx$

$x^2 = 1-t$

$$= \int_0^1 \frac{1}{2}(1-t)^2\sqrt{t}\,dt =$$

$(1-2t+t^2)\sqrt{t}$

$$= \frac{1}{2}\left[\frac{t^{3/2}}{3/2} - 2\cdot\frac{t^{5/2}}{5/2} + \frac{t^{7/2}}{7/2}\right]_0^1 =$$

$$= \frac{1}{2}\left[\frac{2}{3} - \frac{4}{5} + \frac{2}{7}\right] = \frac{35-42+15}{105} = \frac{8}{105}$$

---

**10.63.** OK

$$\int_0^1 x^4\sqrt{1-x^2}\,dx =$$

$x = \sin t,$

$$= \int_0^{\pi/2} \sin^4 t\,\cos^2 t\,dt = \int_0^{\pi/2}(\sin^4 t - \sin^6 t)\,dt$$

$$= \frac{3}{16}\pi - \frac{5}{6}\,\frac{3\pi}{16} = \frac{1}{32}\pi.$$

---

**10.62.** *

$$\int_0^1 x^2\sqrt{1-x^2}\,dx =$$

$x = \sin t, \quad dx = \cos t\,dt$

$$= \int_0^{\pi/2}\sin^2 t\,\cos^2 t\,dt = \int_0^{\pi/2}\frac{1}{4}\sin^2 2t\,dt =$$

$u = 2t, \quad du = 2dt$

$$= \int_0^{\pi}\frac{1}{8}\sin^2 u\,du =$$

$$= \frac{1}{8}\left[\frac{u-\cos u\sin u}{2}\right]_0^{\pi} = \frac{\pi}{16}$$

---

**62.** pokr.

$$\int_0^{\pi/2}\sin^6 t\,dt \underset{PP}{=} \underbrace{\left[-\cos t\sin^5 t\right]_0^{\pi/2}}_{=0} + 5\int\sin^4 t\cos^2 t\,dt$$

$\sin t \quad \sin^5 t$

$-\cos t \quad 5\sin^4 t\cos t$

$$= 5\int\sin^4 t\,dt - 5\int\sin^6 t\,dt$$

$$\Rightarrow \int_0^{\pi/2}\sin^6 dt = \frac{5}{6}\int_0^{\pi/2}\sin^4 t\,dt$$

$$\int_0^{\pi/2}\sin^4 t\,dt = \int_0^{\pi/2}\left(\frac{1-\cos 2t}{2}\right)^2 dt = \int_0^{\pi/2}\left(\frac{1}{4} - \frac{\cos 2t}{2} + \frac{\cos^2 2t}{4}\right)dt$$

$$= \frac{\pi}{8} - \frac{1}{4}\left[\sin 2t\right]_0^{\pi/2} + \frac{1}{8}\left[\frac{2t+\sin 2t\cos 2t}{2}\right]_0^{\pi/2} =$$

$$= \frac{\pi}{8} - 0 + \frac{1}{8}\,\frac{\pi}{2} = \frac{3}{16}\pi.$$

**10.71** ✱ $\int_0^1 \sqrt{\frac{x+1}{x+0}}\,dx$

$t = \sqrt{\frac{x+1}{x+0}}$, $x = \frac{-1}{1-t^2} = \frac{1}{t^2-1}$

$\frac{dx}{dt} = \frac{-2t}{(t^2-1)^2}$

$= \int_{\sqrt{2}}^{+\infty} t \cdot \frac{2t}{(t^2-1)^2}\,dt =$

$= \int_{\sqrt{2}}^{+\infty} \frac{1}{2}\left(\frac{1}{t-1} + \frac{1}{(t-1)^2} + \frac{-1}{t+1} + \frac{1}{(t+1)^2}\right) dt$

$2t^2 = A(t-1)(t+1)^2 + B(t+1)^2 + C(t+1)(t-1)^2 + D(t-1)^2$

$t=1 \Rightarrow B = \frac{1}{2}$, $A + C = 0$ $t^3$

$t=-1 \Rightarrow D = \frac{1}{2}$, $-A+B+C+D = 0$ abs

$2C = -B-D = -1 \Rightarrow C = -\frac{1}{2}$

$A = \frac{1}{2}$

$= \frac{1}{2}\left[\ln\left|\frac{t-1}{t+1}\right| - \frac{2t}{(t^2-1)}\right]_{\sqrt{2}}^{\infty} = -\frac{1}{2}\ln\left|\frac{\sqrt{2}-1}{\sqrt{2}+1}\right| + \sqrt{2}$

$= \frac{1}{2}\ln\left|\frac{\sqrt{2}+1}{\sqrt{2}-1}\right| + \sqrt{2} = \frac{1}{2}\cdot 2\cdot\ln(1+\sqrt{2})^2 + \sqrt{2} =$

$= 1\ln(1+\sqrt{2}) + \sqrt{2}$

$\frac{10.75}{OK}$ $\int_4^{+\infty} \frac{1}{x^2}\sqrt{\frac{x-2}{x-4}}\,dx =$

$t = \sqrt{\frac{x-2}{x-4}} \Rightarrow \boxed{x = \frac{dt^5 - b}{a - ct^3}}$

$\Rightarrow x = \frac{-4t^2+2}{1-t^2}$

$\frac{dx}{dt} = \frac{-8t(1-t^2) + 2t(-4t^2+2)}{(1-t^2)^2} =$

$= \frac{-4t}{(1-t^2)^2}$

$= \int_1^{+\infty} \frac{\cancel{(1-t^2)^2}}{(-4t^2+2)^2} \cdot t \cdot \frac{4t}{\cancel{(1-t^2)^2}}\,dt$

$\cancel{= \int_1^{+\infty} \frac{1}{8} \frac{8t}{(4t^2-2)^2}\,dt = \left[ \frac{1-\frac{1}{4}}{4t^2-2} \right]_1^{\infty} =}$

$\cancel{= \frac{1}{4} \cdot \frac{1}{2} = \frac{1}{8}}$

$= \int_1^{+\infty} \frac{4t^2}{(4t^2-2)^2}\,dt = \int_1^{+\infty} \frac{4t^2\,dt}{(2t-\sqrt{2})^2(2t+\sqrt{2})^2}$

$\cancel{\frac{At+B}{4t^2-2}}$ $\frac{A}{2t-\sqrt{2}} + \frac{B}{(2t-\sqrt{2})^2} + \frac{C}{\ldots}$

$4t^2 = A(2t-\sqrt{2})(2t+\sqrt{2})^2 + B(2t+\sqrt{2})^2 + C(2t-\sqrt{2})^2(2t+\sqrt{2}) + D(2t-\sqrt{2})^2$

$t = \frac{1}{2}\sqrt{2} \Rightarrow 8B = 2 \Rightarrow \underline{B = \frac{1}{4}}$

$t = -\frac{1}{2}\sqrt{2} \Rightarrow 8D = 2 \Rightarrow \underline{D = \frac{1}{4}}$

$8A + 8C = 0 \Rightarrow A = -C$

$-2\sqrt{2}A + 2B + 2\sqrt{2}C + 2D = 0$

$4\sqrt{2}C = -1$

$C = -\frac{1}{4\sqrt{2}}$

$A = \frac{1}{4\sqrt{2}}$

$= \left[ \frac{1}{4\sqrt{2}} \ln \frac{|2t-\sqrt{2}|}{|2t+\sqrt{2}|} - \frac{1}{8}\frac{1}{(2t-\sqrt{2})} - \frac{1}{8}\frac{1}{(2t+\sqrt{2})} \right]_1^{\infty} =$

$= \frac{1}{8\sqrt{2}} \ln \frac{2+\sqrt{2}}{2-\sqrt{2}} + \frac{1}{8}\left(\frac{1}{2-\sqrt{2}} + \frac{1}{2+\sqrt{2}}\right) = \frac{\sqrt{2}}{16}\ln(3+2\sqrt{2})$

$4+4\sqrt{2}+2$

$+ \frac{1}{4}$

**89.** $\int_0^{\pi/2} \frac{\sin x}{\cos^2 x + 3\cos x + 4}\,dx$

$t = \cos x$ ;

$= \int_0^1 \frac{dt}{t^2+3t+4} = \int_0^1 \frac{dt}{\left(t+\frac{3}{2}\right)^2 + \frac{7}{4}} =$

$= \left[\frac{2}{\sqrt{7}} \operatorname{arctg}\left(\frac{2t+3}{\sqrt{7}}\right)\right]_0^1 =$

$= \frac{2}{\sqrt{7}}\left(\operatorname{arctg}\frac{5}{\sqrt{7}} - \operatorname{arctg}\frac{3}{\sqrt{7}}\right) = \frac{2}{7}\sqrt{7}\cdot\operatorname{arctg}\frac{\frac{2}{\sqrt{7}}}{1+\frac{15}{7}} =$

$= \frac{2}{7}\sqrt{7}\operatorname{arctg}\frac{\sqrt{7}}{11}$

$\Big[\ \operatorname{arctg} x \pm \operatorname{arctg} y = \operatorname{arctg}\frac{x \pm y}{1 \mp xy} =$ ???

$x = \operatorname{tg} p,\ y = \operatorname{tg} q$

~~$\operatorname{tg} p + \operatorname{tg} q =$~~ $\operatorname{tg}(p \pm q) = \frac{\operatorname{tg} p \pm \operatorname{tg} q}{1 \mp \operatorname{tg} p \operatorname{tg} q} =$

$p \pm q = \operatorname{arctg}\left(\frac{x \pm y}{1 \mp xy}\right)\ \Big]$

**88.** $\int_{-\pi/4}^{\pi/4} \frac{dx}{\cos^4 x} =$

$= \int_{-1}^{1} (t^2+1)\,dt = \left[\frac{t^3}{3} + t\right]_{-1}^{1} =$

$t = \operatorname{tg} x \ldots \qquad = \frac{4}{3} + \frac{4}{3} = \underline{\underline{\frac{8}{3}}}.$

**87.** $\int_0^{4\pi} \frac{dx}{\cos x + 2\sin x + 3} = \qquad \int_{-\pi}^{\pi} + \int_{\pi}^{3\pi}$

$t = \operatorname{tg}\frac{x}{2} \qquad dx = \frac{2}{1+t^2}\,dt$

$\sin x = \frac{2t}{1+t^2},\ \cos x = \frac{1-t^2}{1+t^2}$

$= 2\cdot\int_{-\pi}^{\pi} \ldots\, dx = 2\int_{-\infty}^{+\infty} \frac{1\cdot 2\,dt}{1-t^2+4t+3(1+t^2)} =$

$= 4\int_{-\infty}^{+\infty} \frac{dt}{2t^2+4t+4} =$

$= 2\int_{-\infty}^{+\infty} \frac{dt}{(t+1)^2+1} =$

$= 2\left[\operatorname{arctg}(t+1)\right]_{-\infty}^{+\infty} = \underline{\underline{2\pi}}.$

$\underline{95.}$ * $\displaystyle\int_0^{\pi/2} \frac{dx}{1+\operatorname{tg} x}$

$t = \operatorname{tg} x$

$\dfrac{dt}{dx} = \dfrac{1}{\cos^2 x} = (1+\operatorname{tg}^2 x)$

$dx = \dfrac{dt}{1+t^2}$

$$\int_0^{+\infty} \frac{dt}{(1+t)(1+t^2)} =$$

$$= \int_0^{\infty} \left[\frac{1}{2}\frac{1}{1+t} - \frac{1}{2}\frac{t-1}{t^2+1}\right] dt =$$

$\cancel{(At+B)(1+t^2)+C(1+t)}\ 1 = A(1+t^2) + (Bt+C)(1+t)$

$t=-1:\ A = \frac{1}{2},\quad t = i;\ 1 = (Bi+C)(1+i)$

$1 = (-B+C+Bi+Ci)$

$-B = +C = \frac{1}{2}$

$$= \int_0^{\infty} \left[\frac{1}{2}\frac{1}{1+t} - \frac{1}{4}\frac{2t}{t^2+1} + \frac{1}{2}\frac{1}{t^2+1}\right] dt$$

$$= \left[\frac{1}{4}\ln\frac{(1+t)^2}{(t^2+1)} + \frac{1}{2}\operatorname{arctg} t\right]_0^{\infty} = \underline{\underline{\frac{1}{4}\pi}}$$

* $\underline{96.}$ $\displaystyle\int_{-\pi/2}^{\pi/2} \frac{1+\sin x}{1+\cos x}\, dx =$

$\cancel{\displaystyle\int_{-\pi/2}^{\pi/2} \frac{1+\sin x - \cos x - \sin x\cos x}{\sin^2 x}\, dx =}$

$\cancel{= \displaystyle\int_{-\pi/2}^{\pi/2} \left(\frac{1}{\sin^2 x} + \frac{1}{\sin x} - \frac{\cos x}{\sin^2 x} - \operatorname{cotg} x\right)}$

$t = \operatorname{tg}\frac{x}{2},\quad t \in (-1, 1)$

$$\int_{-1}^{1} \frac{1 + \frac{2t}{t^2+1}}{1 + \frac{1-t^2}{t^2+1}} \frac{2\,dt}{t^2+1} =$$

$$= \int_{-1}^{1} \frac{t^2+1+2t}{(t^2+1)\cdot \cancel{2}} \frac{\cancel{2}\,dt}{1} =$$

$$= \int_{-1}^{1} \left(1 + \frac{2t}{t^2+1}\right) dt = \left[t + \ln(t^2+1)\right]_{-1}^{1}$$

$$= 1 + \ln 2 - (-1 + \ln 2) = \underline{\underline{2}}.$$