(1) $x^3 + y^3 - 3xy(x-y)$

$= x^3 + y^3 - 3x^2y + 3xy^2$

$D_f = \mathbb{R}^2$

$\frac{\partial f}{\partial x} = 3x^2 - 6xy + 3y^2$

dce spojité na $\mathbb{R}^2$

D $\exists$

$\frac{\partial f}{\partial y} = 3y^2 - 3x^2 + 6xy$

$D_f(h) = (3x_0^2 - 6x_0y_0 + 3y_0^2)\, h_1$
$+ (3y_0^2 - 3x_0^2 + 6x_0y_0)\, h_2$

(2) $\sin(x^2+y^2)$ $\mathbb{R}^2$

$\frac{\partial f}{\partial x} = \cos(x^2+y^2) \cdot 2x$

dce spoj. na $\mathbb{R}^2$ $\rightsquigarrow$ $D_f(h)$ $\exists$

$\frac{\partial f}{\partial y} = \cos(y^2+x^2) \cdot 2y$

(3) $\ln(x+y)$ $x+y > 0$

$\frac{\partial f}{\partial x} = \frac{1}{x+y}$

$x \wedge y \neq 0$

$\frac{\partial f}{\partial y} = \frac{1}{x+y}$

$D_f(h)$ $\exists$ na ~~D_f~~

(4) $$f(x,y) = \begin{cases} (x^2+y^2)\sin\dfrac{1}{x^2+y^2} & \mathbb{R}^2 \setminus (0,0) \\ 0 & (0,0) \end{cases}$$

• na $\mathbb{R}^2 \setminus (0,0)$

$$\frac{\partial f}{\partial x} = 2x\sin\frac{1}{x^2+y^2} + x^2\cos\frac{1}{x^2+y^2}\cdot 2x + y^2\cos\frac{1}{x^2+y^2}\cdot 2x$$

$\dfrac{\partial f}{\partial y}$ = prohodím x a y

spojité na $\mathbb{R}^2 \setminus (0,0)$

• v bodě $(0,0)$

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$$\begin{cases} e^{-\frac{1}{x^2+y^2}} & \mathbb{R}^2 \setminus (0,0) \\ 0 & (0,0) \end{cases}$$

• $\mathbb{R}^2 \setminus (0,0)$

$$\frac{\partial f}{\partial x} = e^{-\frac{1}{x^2+y^2}} \cdot \frac{+1}{(x^2+y^2)^2} \cdot 2x$$

• $(0,0)$

$$\frac{\partial f}{\partial x} = \lim_{x \to 0} \frac{e^{-\frac{1}{x^2}} - 0}{x}$$

$$= 0$$
stejně

$$\frac{\partial f}{\partial y} = 0 \qquad \text{analogicky}$$

• $D_f(h)$ v 0:

$$\lim_{h \to 0} \frac{e^{-\frac{1}{h_1^2+h_2^2}} - 0}{\sqrt{h_1^2 + h_2^2}} \underset{\text{stejně}}{=} 0$$

$$\lim_{z \to \infty} \frac{e^{-z}}{\frac{1}{\sqrt{z}}} = \lim \frac{\sqrt{z}}{e^z} \overset{l'H}{=} \lim \frac{\frac{1}{2}\frac{1}{\sqrt{z}}}{e^z} = 0$$

$$f(x,y) = |y| \sin x$$

$$D_f = \mathbb{R}^2$$

$$\frac{\partial f}{\partial x}(0,0) = \lim_{x \to 0} \frac{0 \cdot \sin x}{x} = 0$$

$$\frac{\partial f}{\partial y}(0,0) = \lim_{y \to 0} \frac{|y| \cdot \sin 0}{y} = 0$$

$$D_f = \lim_{h \to 0} \frac{|h_2| \sin h_1 - 0 - 0}{\sqrt{h_1^2 + h_2^2}} =$$

$$= \lim \frac{\sin h_1}{h_1} \cdot \frac{h_1 \cdot |h_2|}{\sqrt{h_1^2 + h_2^2}}$$

$$h_1^2 + 2|h_1 h_2| + h_2^2$$

AG nerovnost $\sqrt{h_1^2 \cdot h_2^2} \leq \frac{h_1^2 + h_2^2}{2}$

$$\frac{h_1 \cdot h_2}{\sqrt{h_1^2 + h_2^2}} \leq \frac{1}{4} \frac{(h_1 + h_2)^2}{\sqrt{h_1^2 + h_2^2}} = \sqrt{h_1^2 + h_2^2} \to 0$$

(2) $\cos \sqrt[3]{xy}$ $\quad\mathbb{R}^2$

$$\frac{\partial f}{\partial x} = -\left(\sin \sqrt[3]{xy}\right) \cdot \frac{1}{3} \frac{1}{\sqrt[3]{(xy)^2}} \cdot y$$

$$\frac{\partial f}{\partial y} = -\sin \sqrt[3]{xy} \cdot \frac{1}{3} \cdot \frac{1}{\sqrt[3]{(xy)^2}} \cdot x$$

pro $x \cdot y \neq 0$

$$\frac{\partial f}{\partial x}(0,0) = \lim_{x \to 0} \frac{\cos \sqrt[3]{x \cdot 0} - 1}{x} = 0$$

$$\frac{\partial f}{\partial y}(0,0) = \lim_{y \to 0} \frac{\cos \sqrt[3]{y \cdot 0} - 1}{y} = \lim \frac{0}{y} = 0$$

$D_f(0,0)(h)$ $\exists$ ✓

$$\lim_{h \to 0} \frac{\cos \sqrt[3]{h_1 \cdot h_2} - 1 - 0}{\sqrt{h_1^2 + h_2^2}} =$$

$$= \lim \underbrace{-\frac{1 - \cos \sqrt[3]{h_1 \cdot h_2}}{(h_1 \cdot h_2)^{2/3}}}_{\text{om}} \cdot \frac{(h_1 \cdot h_2)^{2/3}}{\sqrt{h_1^2 + h_2^2}} \overset{\text{om} \cdot 0}{=} 0$$

$$\left(\sqrt{h_1^2 h_2^2}\right)^{2/3} \leq \frac{(h_1^2 + h_2)^{2/3}}{(h_1^2 + h_2^2)^{1/2}} \to 0$$

$$f(x,y) = \sqrt{|x|^3+|y|^3}$$

$\mathbb{R}^2 \setminus (0,0)$

$$\frac{\partial f}{\partial x} = \frac{1}{2} \cdot \frac{1}{\sqrt{|x|^3+|y|^3}} \cdot 3|x|^2 \cdot \operatorname{sgn} x$$

$$\frac{\partial f}{\partial x}(0,0) = \lim_{x \to 0} \frac{\sqrt{|x|^3+|0|^2}}{x} = 0$$

$$\frac{\partial f}{\partial y}(0,0) = 0$$

$D_f(0,0)\cdot(h)$ $\exists$ ✓

$$\lim_{h \to 0} \frac{\sqrt{|h_1|^3+|h_2|^3} - 0}{\sqrt{h_1^2+h_2^2}} = \lim \sqrt{\frac{|h_1|^3+|h_2|^3}{h_1^2+h_2^2}} = 0$$

$$|h_1|^3 + |h_2|^3 \leq \underset{\overset{\downarrow}{0}}{(h_1+h_2)}(h_1^2+h_2)^2$$

$f(x,y) = \sqrt[3]{xy}$ $\qquad \mathbb{R}^2$

na $\mathbb{R}^2 \setminus (0,0)$

$$\frac{\partial f}{\partial x} = \frac{1}{3} \frac{1}{(xy)^{2/3}} y$$

$$\frac{\partial f}{\partial x}(0,0) = \lim_{x \to 0} \frac{\sqrt[3]{x \cdot 0}}{x} = 0$$

$Df(0,0)(h)$

$$\lim_{h \to 0} \frac{\sqrt[3]{h_1 \cdot h_2} - 0}{\sqrt{h_1^2 + h_2^2}}$$ neexistuje, neb

pro $h_1 = h_2$ máme

$$\lim_{h_1 \to 0} \frac{h_1^{2/3}}{\sqrt{2 h_1^2}} = \infty \neq 0$$

$h_1 = 0 \to \lim = 0$