$$I(a,b) = \int_0^\infty \frac{\operatorname{arctg} ax - \operatorname{arctg} bx}{x}\,dx$$

$\boxed{\text{Fubinka}}$

- $a, b > 0, \qquad b < a$

$$I(a,b) = \int_0^\infty \left[\frac{\operatorname{arctg} yx}{x}\right]_b^a dx = \int_0^\infty \int_b^a \frac{1}{1+y^2x^2} \cdot \frac{x}{x}\,dy\,dx$$

$$\overset{\text{Fub}}{=} \int_b^a \int_0^\infty \frac{1}{1+y^2x^2}\,dx\,dy = \int_b^a \frac{1}{y}\left[\operatorname{arctg} yx\right]_0^\infty dy$$

$$= \int_b^a \frac{1}{y} \cdot \frac{\pi}{2}\,dy = \frac{\pi}{2}\left[\ln y\right]_b^a = \underline{\underline{\frac{\pi}{2} \ln \frac{a}{b}}}$$

- $a, b > 0, \qquad a < b$

$$\int_0^\infty \left[-\frac{\operatorname{arctg} yx}{x}\right]_a^b dx = -\int_0^\infty \int_a^b \frac{1}{1+y^2x^2}\,dy\,dx =$$

$$= -\int_a^b \frac{1}{y}\frac{\pi}{2}\,dy = -\frac{\pi}{2}\left(\ln b - \ln a\right) = \underline{\underline{-\frac{\pi}{2} \ln \frac{b}{a}}}$$

- $a, b < 0 \qquad b < a$

$$I(a,b) = \dots = \int_b^a \frac{1}{y}\left[\operatorname{arctg} yx\right]_0^\infty dy = \int_b^a \frac{1}{y}\left(-\frac{\pi}{2}\right) =$$

$$= \underline{\underline{-\frac{\pi}{2} \ln \frac{a}{b}}}$$

- $a, b < 0 \qquad a < b$

$$I(a,b) = \dots = -\int_a^b \frac{1}{y}\left(-\frac{\pi}{2}\right)dy = \underline{\frac{\pi}{2} \ln \frac{b}{a}}$$

- $a = b \qquad I(a,b) = 0$

- $a \leq 0 \quad b > 0$
- $a > 0 \quad b \leq 0$

$\Big\}$ $\int$ diverguje

$a<b$

- $a \leq 0 < b$

$$\int_0^\infty \left[-\frac{\operatorname{arctg} yx}{x} \right]_a^b dx = -\int_0^\infty \int_a^b \frac{1}{1+y^2x^2}\, dy\, dx =$$

$$= -\int_a^0 \int_0^\infty \frac{1}{1+y^2x^2}\, dx\, dy + -\int_0^b \int_0^\infty \frac{1}{1+y^2x^2}\, dx\, dy$$

$$= -\int_a^0 \frac{1}{y} \left[\operatorname{arctg} yx \right]_0^\infty dy - \int_0^b \frac{1}{y} \left[\operatorname{arctg} yx \right]_0^\infty dy$$

$$= -\int_a^0 -\frac{\pi}{2} \cdot \frac{1}{y}\, dy - \int_0^b \frac{\pi}{2} \cdot \frac{1}{y}\, dy =$$

$$= -\int_a^b \frac{\pi}{2} \frac{1}{|y|}\, dy = \underline{\underline{-\infty}}$$

- $b \leq 0 < a$

$$\int_0^\infty \left[\frac{\operatorname{arctg} yx}{x} \right]_b^a = \int_b^0 \frac{1}{y} \left[\operatorname{arctg} yx \right]_0^\infty dy + \int_0^a \frac{1}{y} \left[\operatorname{arctg} yx \right]_0^\infty dy$$

$$= \int_b^0 -\frac{1}{y} \frac{\pi}{2}\, dy + \int_0^a \frac{\pi}{2} \frac{1}{y}\, dy = \int_b^a \frac{\pi}{2} \frac{1}{|y|}\, dy = \underline{\infty}$$