1. písemka

Jméno:

1. Prohoďte pořadí integrace

$$\int_{-2}^{2}\int_{x^2-4}^{-2x+4} f(x,y)\mathrm{d}y\,\mathrm{d}x$$

2. Převeďte na trojný integrál

$$\int_M xy\,\mathrm{d}x\mathrm{d}y\mathrm{d}z,$$

kde $M = \{0 \le z \le y^2;\ x^2+y^2 \le 1;\ y \ge 0\}$

3. Spočtěte

$$\int_M (x+y)\,\mathrm{d}x\mathrm{d}y,$$

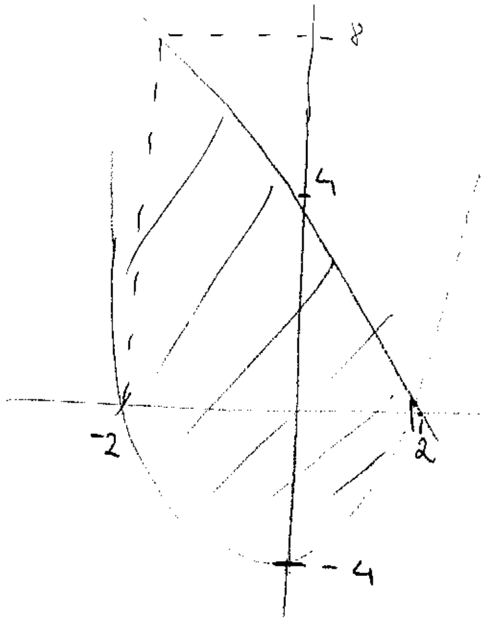
kde $M = \{x^2 + y^2/4 \le 4x\}$.

4. Spočtěte

$$\int_M (x+1)\,\mathrm{d}x\mathrm{d}y,$$

kde M je ohraničena křivkami $y = 2x$, $2y = x$ a $y = 2$.

(1) $$\int_{-2}^{2} \int_{x^2-4}^{-2x+4} f(x,y)\, dy\, dx$$



$$y = x^2 - 4 \qquad \pm\sqrt{y+4} = x$$

$$\int_{-4}^{0} \int_{-\sqrt{y+4}}^{\sqrt{y+4}} f(x,y)\, dx\, dy$$

$y \qquad x$

$$y = -2x + 4 \qquad \frac{y-4}{-2} = x$$

$$+ \int_{0}^{8} \int_{-2}^{-\frac{y}{2}+2} f(x,y)\, dx\, dy$$

② $\int\limits_M xy \, dx\,dy\,dz$

$$0 \le z \le y^2$$
$$x^2+y^2 \le 1$$
$$y \ge 0$$

$x = r\cos\varphi \qquad J = r$

$y = r\sin\varphi$

$z = z$

$$0 \le z \le r^2\sin^2\varphi$$
$$0 \le r^2 \le 1$$
$$0 \le r\sin\varphi$$
$$0 \le \sin\varphi \qquad \varphi \in (0, \pi)$$

$$\int\limits_0^{\pi} \int\limits_0^{1} \int\limits_0^{r^2\sin^2\varphi} r\cos\varphi \, r\sin\varphi \, r \, dz\,dr\,d\varphi$$

$\varphi \quad r \quad z$

(3) $\int x+y \, dx\,dy$

$x^2 + \cancel{\frac{y^2}{4}} \leq 4x$

$(x-2)^2 + \frac{y^2}{4} \leq 4$

$x = 2 + r\cos\varphi$

$y = 2r\sin\varphi$

$J = 2r$

$r^2\cos^2\varphi + r^2\sin^2\varphi \leq 4$

$r^2 \leq 4$

$r \in (0,2)$

$\varphi \in (0, 2\pi)$

$$\int_0^{2\pi}\int_0^2 (2 + r\cos\varphi + 2r\sin\varphi)2r \, dr\,d\varphi$$

$$= 2\int_0^{2\pi} \left[2r + \frac{1}{2}r^2\cos\varphi + r^2\sin\varphi\right]_0^2 d\varphi$$

$$= 2\int_0^{2\pi} 4 + 2\cos\varphi + 4\sin\varphi \quad d\varphi$$

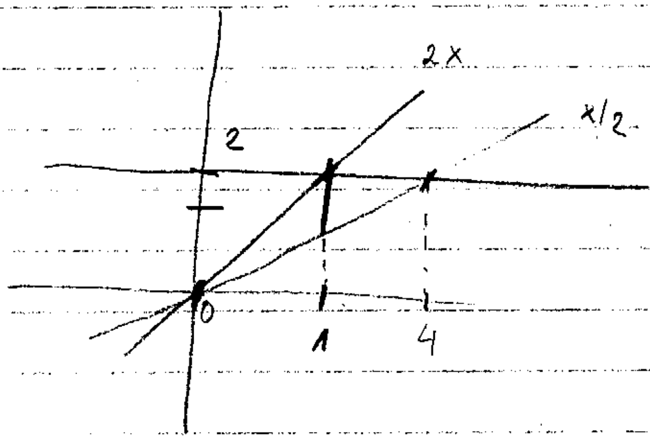
$$= 2\left[4\varphi + 2\sin\varphi + (-4)\cos\varphi\right]_0^{2\pi}$$

$$= 2\cdot 4\cdot 2\pi = \underline{\underline{16\pi}}$$

(4) $\int\limits_M (x+1)\,dx\,dy$

$y = 2x$
$2y = x$
$y = 2$

$\int\limits_0^1 \int\limits_{x/2}^{2x} (x+1)\,dy\,dx$



$\int\limits_1^4 \int\limits_{x/2}^{2} (x+1)\,dy\,dx$

$\int\limits_0^1 (x+1)\left(2x - \frac{x}{2}\right) dx = \int 2x^2 - \frac{x^2}{2} + 2x - \frac{x}{2}\,dx =$

$= \int \frac{3}{2}x^2 + \frac{3}{2}x\,dx = \left[\frac{3}{2}\cdot\frac{1}{3}x^3 + \frac{3}{2}\cdot\frac{1}{2}x^2\right]_0^1 dx$

$= \frac{1}{2} + \frac{3}{4} = \frac{10}{12}$

$\int\limits_1^4 \int\limits_{x/2}^{2} (x+1)\,dy\,dx = \int (x+1)\left(2 - \frac{x}{2}\right) dx = \int 2x + 2 - \frac{x^2}{2} - \frac{x}{2}\,dx$

$= \left[x^2 + 2x - \frac{1}{2}\cdot\frac{1}{3}x^3 - \frac{1}{2}\cdot\frac{1}{2}x^2\right]_1^4 =$

$= 16 + 8 - \frac{1}{6}64 - \frac{1}{4}16 - 1 - 2 + \frac{1}{6} + \frac{1}{4}$

dohromady $= 32 - \frac{32}{3} - 3 - 4 + \frac{1}{6} + \frac{1}{4} = 25 + \frac{128 + 2 + 3}{12}$

$= 25 + \frac{133}{12}$ alebo $25 + \frac{143}{12}$