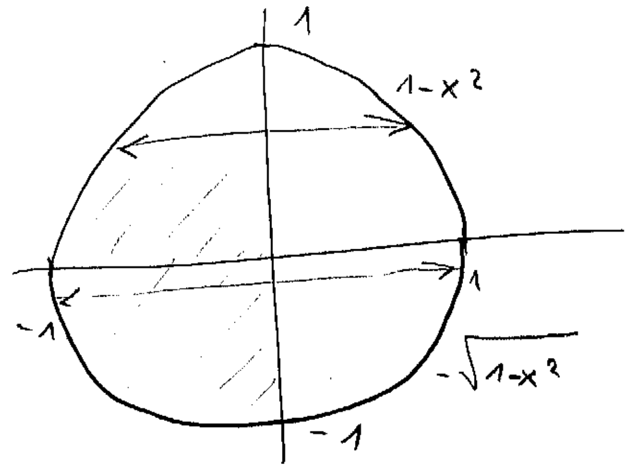
(1) $\int_{-1}^{1} \int_{-\sqrt{1-x^2}}^{1-x^2} f(x,y)\, dy\, dx$




$y \in (-1, 1)$

(a)

$y = -\sqrt{1-x^2}$

$-y = \sqrt{1-x^2}$

$(-y)^2 = 1-x^2$

$x^2 = 1-y^2$

$x = \pm\sqrt{1-y^2}$

(b)

$y = 1-x^2$

$x^2 = 1-y$

$x = \pm\sqrt{1-y}$

$\int_{-1}^{0} \int_{-\sqrt{1-y^2}}^{\sqrt{1-y^2}} f(x,y)\, dx\, dy + \int_{0}^{1} \int_{-\sqrt{1-y}}^{\sqrt{1-y}} f(x,y)\, dx\, dy$

(2) $\int\limits_M z \, dx\, dy\, dz$

$M: \quad 0 \le z \le \sqrt{x^2+y^2}$

$x^2+y^2-4y \le 0$

$x^2 + (y-2)^2 \le 4$

válcové $\quad x = r\cos\varphi$

$y = r\sin\varphi$

$z = z$

pak: $\quad z \le \sqrt{r^2}$

$0 \le z \le r$

- $r^2 \le 4r\sin\varphi$

$0 \le r \le 4\sin\varphi$

- $0 \le 4\sin\varphi$

$\to \varphi \in (0, \pi)$

$$\int\limits_{\substack{0\\ \varphi}}^{\pi} \int\limits_{\substack{0\\ r}}^{4\sin\varphi} \int\limits_{\substack{0\\ z}}^{r} z \cdot r \; dz\, dr\, d\varphi =$$

(3)

$$\int_M x \, dx\, dy \qquad M: (x-2)^2 + \frac{(y-1)^2}{4} \le 1$$

$x = 2 + r\cos\varphi$

$y = 1 + 2r\sin\varphi$

pak $r^2\cos^2\varphi + \frac{4r^2\sin^2\varphi}{4} \le 1$

$r^2 \le 1$

$r \in (0,1)$ ; $\varphi \in (0, 2\pi)$

$J = 2r$

$$\int_0^{2\pi}\int_0^1 (2 + r\cos\varphi) 2r \, dr\, d\varphi$$

$$= \int_0^{2\pi}\int_0^1 4r + 2r^2\cos\varphi \, dr\, d\varphi =$$

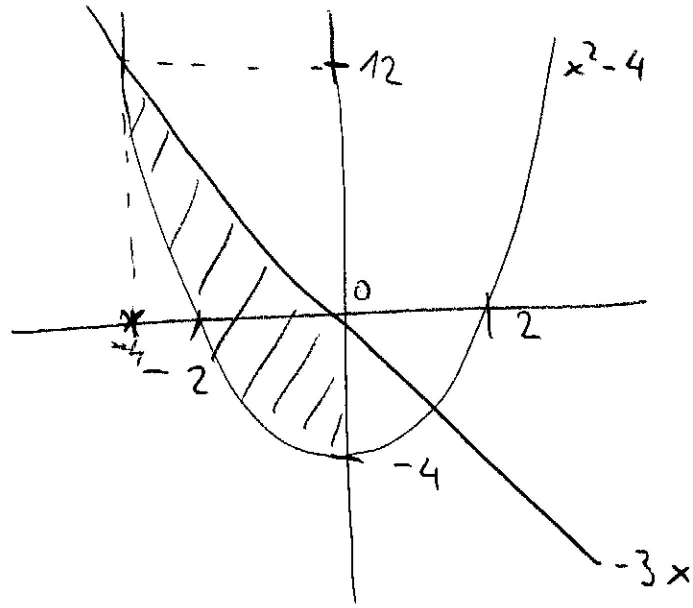
$$= \int_0^{2\pi} \left[2r^2 + \frac{2}{3}r^3\cos\varphi\right]_0^1 d\varphi$$

$$= \int_0^{2\pi} 2 + \frac{2}{3}\cos\varphi \, d\varphi = \left[2\varphi + \frac{2}{3}\sin\varphi\right]_0^{2\pi} d\varphi =$$

$$= \underline{\underline{4\pi}}$$

(4)

$$\iint_M (x+1)\,y \; dx\,dy$$




M: $\quad y = x^2 - 4$

$\quad\; y = -3x$

$\quad\; x = 0$

$\quad\; x \le 0$

$$\int_{-4}^{0}\int_{x^2-4}^{-3x} (x+1)\,y \; dy\,dx$$

$x \quad y$

$x^2 - 4 = -3x$

$x^2 + 3x - 4 = 0$

$(x+4)(x-1) = 0$

$$= \int_{-4}^{0} (x+1)\,\frac{1}{2}\left[y^2\right]_{x^2-4}^{-3x} dx = \int_{-4}^{0} \frac{1}{2}(x+1)\left[9x^2 - (x^2-4)^2\right] dx =$$

$$= \frac{1}{2}\int_{-4}^{0} (x+1)(9x^2 - x^4 - 16 + 8x^2)\, dx =$$

$$= \frac{1}{2}\int_{-4}^{0} 9x^3 - x^5 - 16x + 8x^3 + 9x^2 - x^4 - 16 + 8x^2 \, dx =$$

$$= \frac{1}{2}\left[\frac{9}{4}x^4 - \frac{1}{6}x^6 - 8x^2 + \frac{8}{4}x^4 + \frac{9}{3}x^3 - \frac{1}{5}x^5 - 16x + \frac{8}{3}x^3\right]_{-4}^{0}$$

$$= \underline{\underline{-\frac{1376}{15}}}$$