

① Najedite funkci  $f = u(x, y) + i v(x, y)$  tak-že,  $z_0$

$f$  je holomorfní ve své definiční oblasti,

$$f(0) = 0$$

$$a \quad u(x, y) = e^x(x \cos y - y \sin y)$$

$$\frac{\partial u}{\partial x} = e^x(x \cos y - y \sin y) + e^x(\cos y) = \frac{\partial v}{\partial y}$$

$$\frac{\partial u}{\partial y} = e^x x (-\sin y) - e^x(1 \cdot \sin y + y \cos y) = -\frac{\partial v}{\partial x}$$

$$v = e^x x \sin y - e^x(\sin y - y \cos y) + e^x \sin y + k(x)$$

$$\frac{\partial v}{\partial x} = e^x(x+1)(\sin y) + e^x y \cos y + k'(x) + \dots$$

$$\rightarrow k'(x) = 0$$

$$k(x) = c$$

celkem  $v = e^x \sin y + e^x y \cos y + c$

podle podmínek

$$f(0) = e^0(0 \cdot \cos 0 - 0 \cdot \sin 0) + i(e^0 \cdot 0 \sin y + e^0 \cdot 0 \cos y + c) = 0 \rightarrow c = 0$$

celkem  $f(x + iy) = e^x(x \cos y - y \sin y) + i(e^x x \sin y + e^x y \cos y)$

③  $\text{Argzente } w \in \mathbb{C}$

$$e^w = -2$$

$$e^{x+iy} = e^x (\cos y + i \sin y)$$

$$-2 = 2 (\cos(\pi + 2k\pi) + i \sin(\pi + 2k\pi))$$

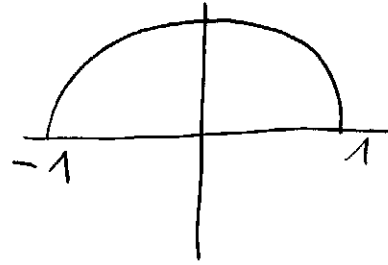
$$\rightarrow e^x = 2 \quad x = \ln 2$$

$$y = \pi + 2k\pi \quad k \in \mathbb{Z}$$

$$\text{Letken } w = \ln 2 + i(\pi + 2k\pi), \quad k \in \mathbb{Z}$$

② Spoctete  $\int_C |z| dz$ , kde  $C$  je horní polokružnice z bodu  $1$  do  $-1$

$$|z| = \sqrt{x^2 + y^2}$$



$$\int_0^{\pi} \sqrt{\cos^2 t + \sin^2 t} (-\sin t + i \cos t) dt$$

$$= \int_0^{\pi} -\sin t + i \cos t dt$$

$$= [\cos t + i \sin t]_0^{\pi}$$

$$= 1 - (-1) + i0 = \underline{\underline{-2}}$$

$t \in [0, \pi]$

$$x(t) = \cos t$$

$$y(t) = \sin t$$

$$x'(t) = -\sin t$$

$$y'(t) = \cos t$$