$$\lim_{x\to\infty} \frac{\ln^\alpha x}{x^\beta} = 0 \qquad \alpha, \beta > 0$$

- $\alpha, \beta \in \mathbb{N}$,

$$\lim_{x\to\infty} \frac{\ln^\alpha x}{x^\beta} \overset{L'H}{\underset{\frac{\infty}{\infty}}{=}} \lim_{x\to\infty} \frac{\alpha \ln^{\alpha-1} x \cdot \frac{1}{x}}{\beta x^{\beta-1}}$$

$$= \lim_{x\to\infty} \frac{\alpha \ln^{\alpha-1} x}{\beta x^\beta} \overset{L'H}{\underset{\frac{\infty}{\infty}}{=}} \lim_{x\to\infty} \frac{\alpha(\alpha-1)\ln^{\alpha-2} x}{\beta^2 x^\beta} = \overset{L'H}{\ldots} \underset{\text{opakovaně}}{=}$$

$$= \lim_{x\to\infty} \frac{\alpha!}{\beta^\alpha x^\beta} = \underline{\underline{0}}$$

- $\alpha, \beta \in \mathbb{R}$ stejně $\to$ až do okamžiku, kdy $\alpha - k < 0$

$$\lim_{x\to\infty} \frac{\ln^\alpha x}{x^\beta} \overset{L'H}{\underset{\frac{\infty}{\infty}}{=}} \lim_{x\to\infty} \frac{\alpha \ln^{\alpha-1} x}{\beta x^\beta} = \overset{L'H}{\ldots} =$$

$$\lim_{x\to\infty} \frac{\alpha(\alpha-1)\ldots(\alpha-k+1) \overbrace{\ln^{\alpha-k} x}^{\to 0}}{\beta^k \underbrace{x^\beta}_{\to\infty}} = \underline{\underline{0}}$$