(1) $\lim\limits_{n\to\infty} \left(\frac{\pi}{2} - \arctan 5n\right) \cdot \cot\frac{3}{n} = \frac{1}{15}$

$\underbrace{\left(\frac{\pi}{2} - \underset{\frac{\pi}{2}}{\arctan 5n}\right)}_{\to 0} \cdot \underbrace{\cot\frac{3}{n}}_{\infty}$ $\qquad \frac{3}{n} \to 0+$ $\qquad$ nedef

Heine $\quad x_n = n \quad n\to\infty \quad n \neq \infty$

$$\lim_{x\to\infty} \frac{\frac{\pi}{2} - \arctan 5x}{\sin\frac{3}{x}} \cdot \underbrace{\cos\frac{3}{x}}_{1} = \lim_{x\to\infty} \underbrace{\cos\frac{3}{x}}_{1} \cdot \underbrace{\frac{\frac{3}{x}}{\sin\frac{3}{x}}}_{1} \cdot \frac{\frac{\pi}{2} - \arctan 5x}{\frac{3}{x}}$$

$= (*)$

$$\lim_{x\to\infty} \frac{\pi/2 - \arctan 5x}{\frac{3}{x}} \overset{L'H}{\underset{\frac{0}{0}}{=}} \lim_{x\to\infty} \frac{-\frac{1}{1+25x^2}\cdot 5}{-3\cdot\frac{1}{x^2}} = \lim \frac{5}{3} \cdot \frac{x^2}{1+25x^2}$$

$$= \lim_{x\to\infty} \frac{5}{3} \cdot \frac{\cancel{x^2}}{\cancel{x^2}} \cdot \frac{1}{25 + 1/x^2} = \frac{1}{15}$$

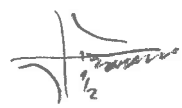
$(*) \overset{\text{VOAL}}{=} 1 \cdot 1 \cdot \frac{1}{15} = \underline{\underline{\frac{1}{15}}}$

VOLSF (1) $f(y) = \cos y \qquad g(x) = \frac{3}{x} \qquad \lim\limits_{x\to\infty} \frac{3}{x} = 0 \qquad \lim\limits_{y\to 0} \cos y = 1$

(S) cos spoj v 0

(2) $f(y) = \frac{y}{\sin y} \qquad \lim\limits_{y\to 0} \frac{y}{\sin y} = 1$

(P) $\frac{3}{x} \neq 0$ na $P(\infty; 2)$



(2)

$$\lim_{x\to 0+} \left(1-\sqrt{\arcsin x}\right)^{\frac{1}{\sqrt[4]{1-\cos x}}}$$

typ "1^{∞}"

$$\lim_{x\to 0+} e^{\frac{1}{\sqrt[4]{1-\cos x}}\ln\left(1-\sqrt{\arcsin x}\right)} =$$

$$\lim_{x\to 0+} \frac{\ln\left(1-\sqrt{\arcsin x}\right)}{\sqrt[4]{1-\cos x}} = \lim_{x\to 0+} \frac{\ln\left(1-\sqrt{\arcsin x}\right)}{-\sqrt{\arcsin x}} \cdot \underbrace{\frac{-\sqrt{\arcsin x}}{\sqrt{x}}}_{-\sqrt{1}} \cdot \frac{\sqrt{x}}{\sqrt[4]{1-\cos x}}$$

$$\overset{\text{VOAL}}{=} 1\cdot(-1)\cdot\sqrt[4]{2} = \underline{\underline{-\sqrt[4]{2}}}$$

$$\lim_{x\to 0+} \frac{\sqrt{x}}{\sqrt[4]{1-\cos x}} = \lim_{x\to 0+} \sqrt[4]{\frac{x^2}{1-\cos x}} = \sqrt[4]{2}$$

VOLS= (1) $f(y)=\sqrt[4]{y}$ $\qquad \lim_{y\to 2} \sqrt[4]{y} = \sqrt[4]{2}$ (S) $\sqrt[4]{y}$ spoj. v 2

$g(x) = \frac{x^2}{1-\cos x}$ $\qquad \lim_{x\to 0} \frac{x^2}{1-\cos x} = 2$

(2) $f(y)=\sqrt{y}$ $\qquad \lim_{y\to 1} \sqrt{y} = 1$ (S) $\sqrt{y}$ spoj. v 1

$g(x)= \frac{\arcsin x}{x}$ $\qquad \lim_{x\to 0+} \frac{\arcsin x}{x} = 1$

(3) $f(y)=\sqrt{y}$ $\qquad \lim_{y\to 0+} \sqrt{y} = 0$ (S) $\sqrt{y}$ spoj. v 0 zprava

$g(x) = \arcsin x$ $\qquad \lim_{x\to 0+} \arcsin x = 0+$

(4) $f(y) = \frac{\ln(1+y)}{y}$ $\qquad \lim_{y\to 0} \frac{\ln(1+y)}{y} = 1$ (P) $\sqrt{\arcsin x} \neq 0$ na $P_+(0, \frac{1}{2})$

$g(x) = -\sqrt{\arcsin x}$ $\qquad \lim_{x\to 0+} -\sqrt{\arcsin x} = 0$

(3) $\underbrace{\sum_{n=2}^{\infty} \arctan\left(\frac{1}{n} - \frac{1}{\sqrt{1+n^2}}\right) \cdot \frac{\sqrt[3]{n+1} - \sqrt[3]{n}}{\sin\left(\frac{1}{n}\right)^{10/3}}}_{a_n \geq 0 \ (*)}$

$$\cdot \quad \frac{1}{n} - \frac{1}{\sqrt{1+n^2}} = \frac{\sqrt{1+n^2} - n}{n\sqrt{1+n^2}} = \frac{1 + \cancel{n^2} - \cancel{n^2}}{\underbrace{n\sqrt{1+n^2}}_{\geq 0}} \cdot \frac{1}{\underbrace{\sqrt{1+n^2} + n}_{\geq 0}}$$

$$\approx \frac{1}{n \cdot n \cdot n} = \frac{1}{n^3}$$

$$\cdot \quad \sqrt[3]{n+1} - \sqrt[3]{n} = \frac{\cancel{n} + 1 - \cancel{n}}{\underbrace{\sqrt[3]{(n+1)^2} + \sqrt[3]{n+1} \cdot \sqrt[3]{n} + \sqrt[3]{n^2}}_{\geq 0}} \approx \frac{1}{\sqrt[3]{n^2}}$$

$$\cdot \quad \sin \frac{1}{n} \approx \frac{1}{n} \qquad \arctan \frac{1}{n^3} \approx \frac{1}{n^3}$$

LSK s $b_n = \frac{1}{n^3} \cdot \frac{1}{\sqrt[3]{n}} \cdot \frac{1}{\left(\frac{1}{n}\right)^{10/3}} = n^{10/3 - 3 - 1/3} = n^0 = 1$

$$\lim_{n\to\infty} \frac{a_n}{b_n} = \lim \frac{\arctan\left(\frac{1}{n} - \frac{1}{\sqrt{1+n^2}}\right)}{\frac{1}{n^3}} \cdot \frac{\sqrt[3]{n+1} - \sqrt[3]{n}}{1/\sqrt[3]{n^2}} \cdot \frac{\frac{1}{\sin\left(\frac{1}{n}\right)^{10/3}}}{1/\left(\frac{1}{n}\right)^{10/3}}$$

$\overset{\text{VOAL}}{=}$

Heine $x_n = n$ $n \to \infty$ $n \neq \infty$ $\forall n \in \mathbb{N}$

$$\lim_{x\to\infty} \frac{\arctan \frac{1}{x(\sqrt{1+x^2})(\sqrt{1+x^2} + x)}}{\frac{1}{x\sqrt{1+x^2}(\sqrt{1+x^2}+x)}} \cdot \frac{\frac{1}{x\sqrt{1+x^2}(\sqrt{1+x^2}+x)}}{\frac{1}{x^3}} \cdot$$

$$\cdot \quad \frac{\frac{1}{\sqrt[3]{(1+x)^2} + \sqrt[3]{1+x}\,\sqrt[3]{x} + \sqrt[3]{x^2}}}{\frac{1}{\sqrt[3]{x^2}}} \cdot \frac{\left(\frac{1}{x}\right)^{10/3}}{\sin\left(\frac{1}{x}\right)^{10/3}} \overset{\text{VOAL}}{=} 1 \cdot \frac{1}{2} \cdot \frac{1}{3} \cdot 1$$

$$= \frac{1}{6} \in (0, \infty)$$

$$\lim_{x\to\infty} \frac{\cancel{x^3}}{\cancel{x}\,\cancel{\sqrt{x^2}}\,\sqrt{1/x^2+1}\cdot \cancel{x}\,\Big(\underbrace{\sqrt{\tfrac{1}{x^2}+1}}_{1}\;+\underbrace{1}_{1}\Big)} \overset{\text{VOAL}}{=} \frac{1}{2}$$

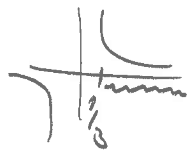
$\Rightarrow$ Vo lim $\sqrt{\ }$ nebo VOLS F

$$\lim_{x\to\infty} \frac{\cancel{\sqrt[3]{x^2}}}{\cancel{\sqrt[3]{x^2}}\Big(\sqrt[3]{(\tfrac{1}{x}+1)^2}+\sqrt[3]{\tfrac{1}{x}+1}+1\Big)} \overset{\text{VOAL}}{=} \frac{1}{3}$$

Vo lim $\sqrt{\ }$ nebo VOLSF

VOLSF (1) $f(y) = \frac{y}{\sin y}$ $\qquad \lim_{y\to 0} \frac{y}{\sin y} = 1$ $\qquad$ (P) $\frac{1}{x} \neq 0$ na $P(\infty, 3)$

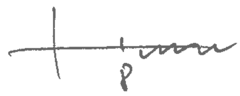
$g(x) = \left(\frac{1}{x}\right)^{10/3}$ $\qquad \lim_{x\to\infty} \left(\frac{1}{x}\right)^{10/3} = 0$



(2) $f(y) = \frac{\arctan y}{y}$ $\qquad \lim_{y\to 0} \frac{\arctan y}{y} = 1$

$g(x) = \frac{1}{x(\sqrt{1+x^2})(\sqrt{1+x^2}+x)}$ $\qquad \lim_{x\to\infty} g(x) = \text{"}\frac{1}{\infty}\text{"} = 0$

(P) $g(x) \neq 0$ na $P(\infty, 1/8)$

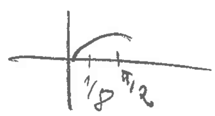


(v čitateli nikdy není 0)

Závěr: z LSK $\sum a_n$ K $\Leftrightarrow$ $\sum b_n$ K

neb $\sum b_n$ D, tak $\sum a_n$ také __DIV__

(*) $\left(\frac{1}{n}\right)^{10/3} \leq \left(\frac{1}{2}\right)^{10/3} \leq \frac{1}{8}$



$\sin\left(\frac{1}{n}\right)^{10/3} \geq 0$

(4) $f(x) = |x| + \arctan\left(\frac{1}{x-3}\right)$

(a) D_f: $x \neq 3$ $\quad x \in (-\infty, 3) \cup (3, \infty)$

f je spoj. na $(-\infty, 3)$ a $(3, \infty)$

složení, součet, podíl spoj.

$f(0) = 0 + \arctan\left(\frac{-1}{3}\right)$

průs. s y ?

(b) $f(-x) = |-x| + \arctan\left(\frac{1}{-x-3}\right) \neq \pm f(x)$

ani sudá, ani lichá, ani period.

(c) $\lim\limits_{x \to +\infty} \underbrace{|x|}_{\infty} + \underbrace{\arctan\Big(\underbrace{\tfrac{1}{x-3}}_{0}\Big)}_{0} \overset{\text{VOLSF}}{=} \infty$

$\lim\limits_{x \to 3+} |x| + \underbrace{\arctan \underbrace{\frac{1}{x-3}}_{1/0+}}_{\pi/2} = 3 + \pi/2$

$\lim\limits_{x \to 3-} \quad = 3 - \pi/2$

(d) $f' = \operatorname{sgn}(x) + \frac{1}{1 + \left(\frac{1}{x-3}\right)^2} \cdot (-1) \cdot \frac{1}{(x-3)^2} =$

$= \operatorname{sgn}(x) + \frac{-1}{1+(x-3)^2} = \begin{cases} 1 - \frac{1}{1+(x-3)^2} & x > 0,\ x \neq 3 \\ -1 - \frac{1}{1+(x-3)^2} & x < 0 \end{cases}$

lim v 3 $\nexists$ (f tam není def), ale pro obrázek se hodí:

$\lim\limits_{x \to 3+} 1 - \frac{1}{1+(x-3)^2} = 0$

$\lim\limits_{x \to 3-} \quad = 0$

$f'_+(0) = \lim_{x\to 0+} 1 - \frac{1}{1+(x-3)^2} = 1 - \frac{1}{10} = 9/10$

$f'_-(0) = \lim_{x\to 0-} -1 - \frac{1}{1+(x-3)^2} = -1 - \frac{1}{10} = -\frac{11}{10}$

$f'(0) \nexists$ V o lim dce (f je spoj.)

(e) $f' = \begin{cases} \frac{1+(x-3)^2-1}{(x-3)^2} \geq 0 & x > 0 \quad x \neq 3 \\ \underbrace{-1 - \frac{1}{1+(x-3)^2}}_{<0} & x < 0 \end{cases}$

0 pro $x = 3$ (ale není tam extrém)

→ f roste na $(0,3)$, $(3,\infty)$

klesá $(-\infty, 0)$

(f) $f'' = -1 \cdot \frac{-1}{(1+(x-3)^2)^2} \cdot 2(x-3) = \frac{2(x-3)}{(1+(x-3)^2)^2}$ $x \neq 0$, $x \neq 3$

(g) $x - 3 = 0$ pro $x = 3$ (ale NENÍ tam inflexe)

		0		3	
f''	$-$		$-$		$+$
f	$\cap$		$\cap$		$\cup$

f je konkávní na $(-\infty, 0)$ a $(0,3)$

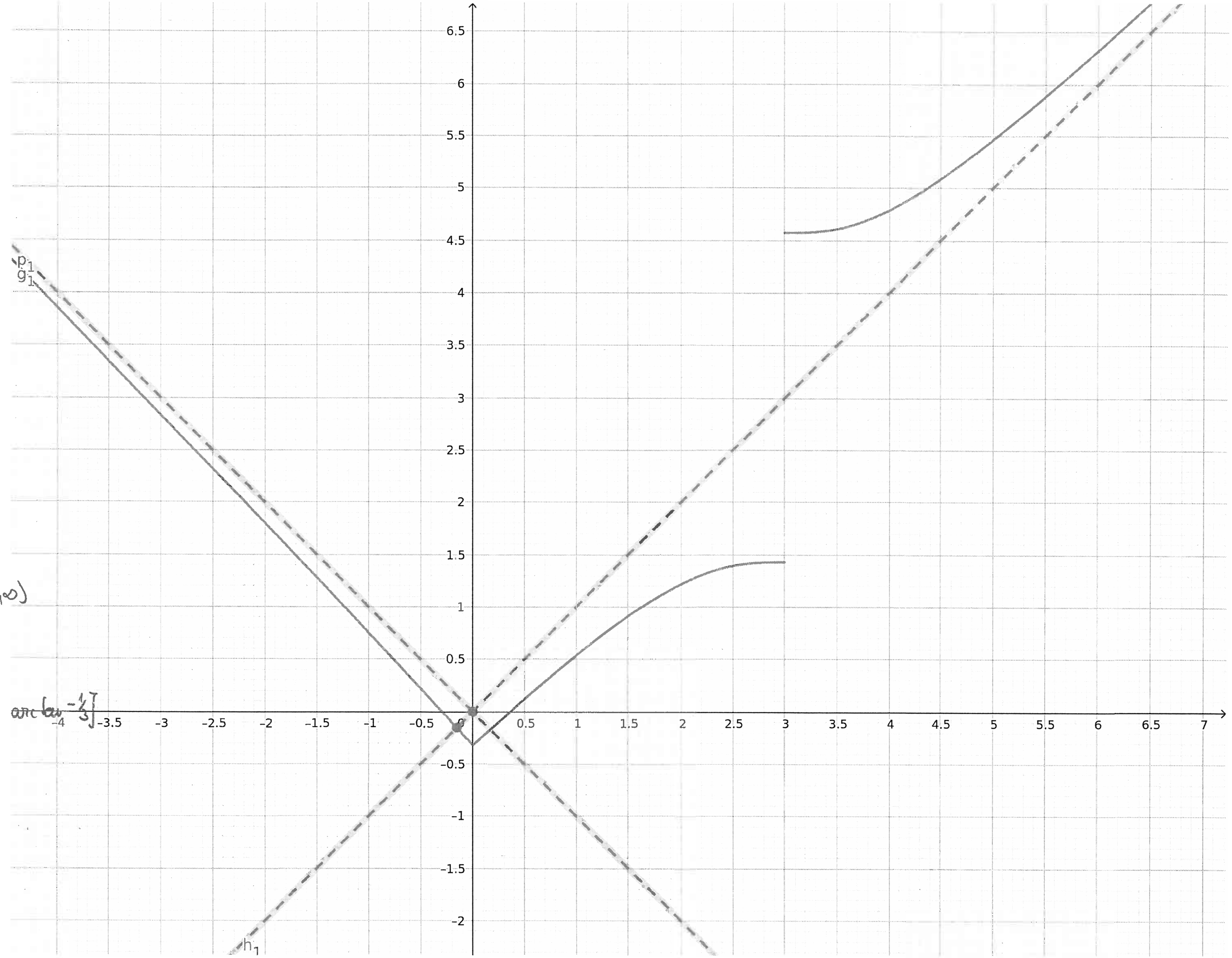
konvexní $(3,\infty)$

(h) $a_1 = \lim_{x\to\infty} \frac{|x|}{x} + \frac{\arctan\left(\frac{1}{x-3}\right)}{x} = 1 + 0 = 1$ $y = x$

$b_1 = \lim_{x\to\infty} x + \arctan\left(\frac{1}{x-3}\right) - 1\cdot x = 0$

$a_2 = \lim_{x\to -\infty} \frac{-x}{x} + \frac{\arctan(\ )}{x} = -1$ $y = -x$

$b_2 = \lim -x + \arctan(\ ) - (-x) = 0$



$H_f = [\arctan \frac{-1}{3}, \infty)$

lok i glob

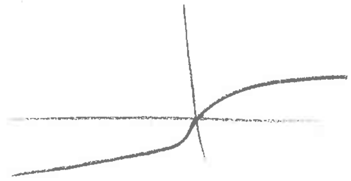
min je u $[0, \arctan -\frac{1}{3}]$

(5) $f(x) = \max\{x + 4\arctan(\sin x) ; x\}$

důležité je znaménko $4\arctan(\sin x)$

→ ale to závisí na znaménku $\sin x$

tedy

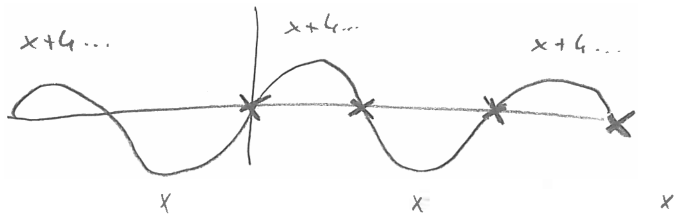


$$f(x) = \begin{cases} x + 4\arctan(\sin x) & \text{pro } \sin x > 0 \quad \text{tedy } x \in (0,\pi) + 2k\pi, \ k \in \mathbb{Z} \\ x & \sin x \leq 0 \quad x \in [-\pi, 0] + 2k\pi \end{cases}$$

f je spoj, protože

$\lim_{x \to 0+} x + 4\arctan(\sin x) = 0 + 0 = 0$

$\lim_{x \to 0-} x = 0$ zbylé body analog.



derivace

$$f'(x) = \begin{cases} 1 + 4 \dfrac{1}{1+\sin^2 x} \cdot \cos x & x \in (0,\pi) + 2k\pi \\ 1 & x \in (-\pi, 0) + 2k\pi \end{cases}$$

v bodech $x = 0 + 2k\pi$ je

$f'_- = 1$ Věta o jednostr. derivaci, f je zjevně spojitá zleva i zprava

$$f'_+ = \lim_{\substack{x \to 0+ \\ +2k\pi \ldots}} 1 + 4 \frac{1}{1+\sin^2 x} \cdot \cos x = 1 + \frac{4}{1+0} 1 = 5$$

der ∄

v bodech $x = -\pi + 2k\pi$ je taktéž Věta

$f'_+ = 1$

$$f'_- = \lim_{x \to (\pi + 2k\pi)-} 1 + \frac{4}{1+\sin^2 x} \cdot \cos x = 1 + \frac{4 \cdot (-1)}{1+0} = -3$$

der ∄