

# Part. 2 Lösung 2

$$\int \frac{x^4 + 4x^3 + 9x^2 + 11x + 4}{(x+3)(x^2+2x+2)^2} dx = \int \frac{A}{x+3} + \frac{Bx+C}{x^2+2x+2} + \frac{Dx+E}{(x^2+2x+2)^2} dx$$

•  $\text{st } P < \text{st } Q$  ✓  
 $4 < 5$

• ( ) ✓

•  $\int \frac{1}{x+3} dx = \ln|x+3|$

$$\int \frac{x}{(x^2+2x+2)^2} dx = \int \frac{1}{2} \cdot \frac{2x+2}{(x^2+2x+2)^2} dx + \int \frac{1}{2} \cdot \frac{-2}{(x^2+2x+2)^2} dx$$

$$\frac{2x+2}{(x^2+2x+2)^2}$$

$$y = x^2 + 2x + 2$$

$$dy = 2x + 2 dx$$

$$\rightarrow \int \frac{1}{y^2} dy = \int \frac{-1}{y} = \frac{-1}{2(x^2+2x+2)}$$

$$- \int \frac{1}{(x^2+2x+2)^2} dx = - \int \frac{1}{[(x+1)^2 + 1]^2} dx$$

$$z = x+1$$

$$dz = 1 dx$$

$$= - \int \frac{1}{(z^2+1)^2} dz = - \frac{z}{2 \cdot 1 \cdot (1+z^2)^1} + \frac{2 \cdot 1 - 1}{2 \cdot 1} \arctan z$$

$$I_2 = I_{1+1}$$

arctanz

$$= \frac{-z}{2(1+z^2)} - \frac{1}{2} \arctan z =$$

$$= \frac{-(x+1)}{2(1+(x+1)^2)} - \frac{1}{2} \arctan(x+1)$$

$x \neq 3$

$$\text{Zusatz} \equiv \ln|x+3| + \frac{-1}{2(x^2+2x+2)} + \frac{-1}{2(x^2+2x+2)} + \frac{-1}{2} \arctan(x+1)$$

$$\int \frac{1}{(1+x^2)^2} dx = \int \frac{1}{(1+\cancel{t^2})^2} \cdot \frac{1}{\cos^2 t} dt =$$

$$x = \cancel{t} t$$

$$t = \arctan x$$

$$dx = \frac{1}{\cos^2 t} dt \neq 0$$

$$\frac{1}{\cos^2 t}$$

$$= \int \cos^2 t \cdot \frac{1}{\cos^2 t} dt = \int \cos^2 t dt =$$

$$= \int \frac{1 + \cos 2t}{2} dt = \frac{t}{2} + \frac{1}{2} \frac{\sin 2t}{2}$$

$$= \frac{t}{2} + \frac{1}{2} \sin t \cos t = \frac{1}{2} \arctan x + \frac{1}{2} \frac{x}{1+x^2}$$

$$x \in \mathbb{R}$$

$$x \in (a, b) \rightarrow$$

2. Vorsubst.

$$\varphi(t) = \arctan t$$

$$(a, b) = \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$$

$$\varphi\left(-\frac{\pi}{2}, \frac{\pi}{2}\right) = \mathbb{R} = (a, b)$$