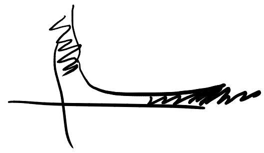
Neabsol. konv. $\int$

$$\int_0^\infty \underbrace{\frac{\arctan x^2}{x^a}\sin(2x)}_{f}\,dx \qquad a>0$$



• problémy: ∞, 0

$$\int_0^\infty = \int_0^{\pi/2} + \int_{\pi/2}^\infty$$

• $\int_0^{\pi/2}$: $1f$ nemění znaménko. AK splývá s K

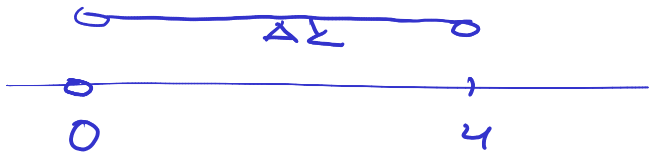
LSK: $g(x)=\dfrac{x^2\cdot 2x}{x^a}$ $\qquad \int_0^{\pi/2} 2\, x^{3-a}\,dx$ K $\iff 3-a>-1$

$\boxed{4>a}$

$g\geq 0$ ✓ $\qquad f, g$ spoj. $(0,\pi/2]$ ✓

$$\lim_{x\to 0+}\frac{f}{g} = \lim_{x\to 0+}\frac{\frac{\arctan x^2}{x^a}\sin(2x)}{\frac{x^2\cdot 2x}{x^a}} = \lim_{x\to 0+}\frac{\arctan x^2}{x^2}\cdot\frac{\sin 2x}{2x} = 1 \in (0,\infty)$$

$\int_0^{\pi/2} f$ K $\iff \int_0^{\pi/2} g$ K $\iff \underline{a<4}$

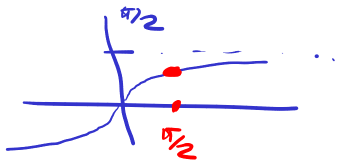


$$\int_{\pi/2}^\infty \frac{\arctan x^2}{x^a}\sin(2x)\,dx$$

• f mění znaménko

AK?: $\displaystyle\int_{\pi/2}^\infty \frac{\arctan x^2}{x^a}\,|\sin(2x)|\,dx$

$\odot \quad \displaystyle\int_{\pi/2}^\infty \frac{|\sin(2x)|}{x^a}\,dx$



• SK (podm.) $\quad \frac{\arctan x^2}{x^a}\,|\sin(2x)| \le \frac{\pi/2 \cdot 1}{x^a}$

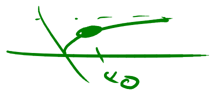
$K \iff a > 1$

ze SK: $\int_{\pi/2}^{\infty} (\quad) \;\; K \;\; \boxed{a>1}$

• $a \le 1$?

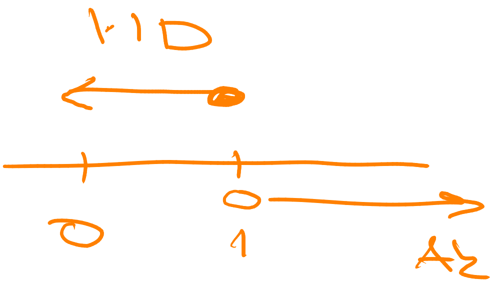
SK $\quad \frac{\arctan x^2}{x^a}\,|\sin 2x| \ge \frac{|\sin(2x)|}{x^a} \quad D$ (tabulka)

$\arctan x^2 \ge 1$ pro $x \in (\frac{\pi}{2}, \infty)$ (od jistého x_0)



ze SK: $a \le 1$

$\int_{\pi/2}^{\infty} \quad D$



$$\int_{\pi/2}^{\infty} \frac{\sin(2x)}{x^a} = \int_{\pi}^{\infty} \frac{1}{2}\frac{\sin y}{\frac{1}{2^a} y^a}\,dy$$

$y = 2x$

$dy = 2\,dx$

$$= \frac{2^a}{2} \int_{\pi}^{\infty} \frac{\sin y}{y^a}\,dy$$

je v tabulce

• $a \in (0, 1]$ NAK ?

$$\int_{\pi/2}^{\infty} \frac{\arctan x^2}{x^a} \sin(2x)\,dx$$

$$\int_{\pi/2}^{\infty} \underbrace{\frac{1}{x^a}}_{g} \underbrace{\sin(2x)}_{f}\,dx \qquad \boxed{\text{Dir}}$$

f, g spoj. $[\pi/2, \infty)$ ✓

F je PF f, $F = -\frac{1}{2}\cos(2x)$ om.

$\lim_{x\to\infty} \frac{1}{x^a} = 0$ ✓ $\quad \frac{1}{x^a}$ monot. ✓

$$\int_{\pi/2}^{\infty} \frac{\sin(2x)}{x^a} \quad K$$

$\boxed{\text{Abel}}$

$$\int_{\pi/2}^{\infty} \underbrace{\operatorname{arctan} x^2}_{g} \; \underbrace{\frac{\sin 2x}{x^a}}_{f}$$

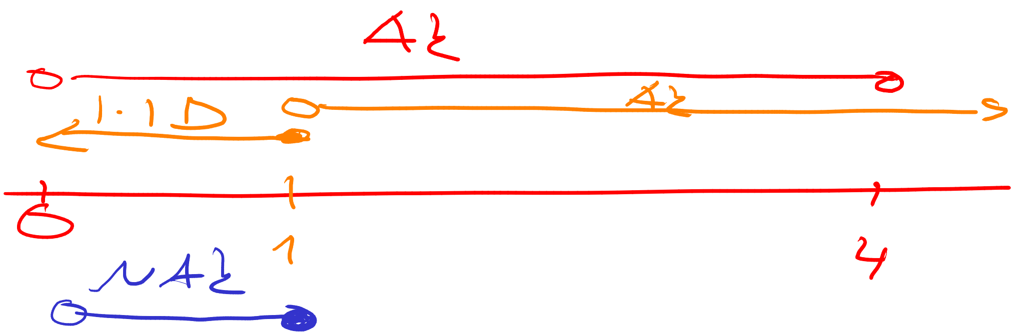
$|\operatorname{arctan} x^2| \le \pi/2$ ✓

g rostoucí

$f \in N(\pi/2, \infty)$ ✓

$$\Rightarrow \int_{\pi/2}^{\infty} \operatorname{arctan} x^2 \, \frac{\sin(2x)}{x^a} \quad K$$

Záver:



$a \in (1,4)$ $\int$ AK

$a \in (0,1]$ $\int$ NAK

$a \in [4,\infty)$ $\int$ D

$$\int_1^\infty \frac{e^x}{e^x} x^a e^{bx}\, dx = \int_e^\infty \frac{1}{y} (\ln y)^a \, y^b \, dy$$

$$= \int_e^\infty y^{b-1} (\ln y)^a \, dy$$

$x = \ln y$

$y = e^x$

$dy = e^x \, dx$

x	1	∞
y	e	∞

~ babalcoü