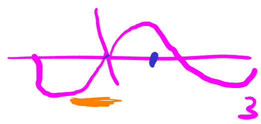
$$\int \frac{\overbrace{\sqrt{9-x^2}}^{f(x)}}{x^2}\,dx \qquad f = \frac{\sqrt{9-x^2}}{x^2} \qquad = \begin{matrix}(0,3) & (-3,0)\\ (a,b) & (a_1,b_1)\\ & (\alpha_1,\beta_1) = (-\tfrac{\pi}{2},0)\end{matrix}$$

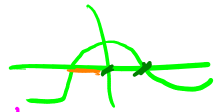
$$x = 3\sin t \qquad \varphi(t) = 3\sin t \qquad (\alpha,\beta) = (0,\tfrac{\pi}{2})$$

$$dx = 3\cos t\,dt \qquad \varphi'(t) = 3\cos t \qquad \varphi(0,\tfrac{\pi}{2}) = (0,3)$$

$$t = \arcsin\frac{x}{3} \qquad \varphi^{-1}(x) = \arcsin\frac{x}{3} \qquad 3\cos t \neq 0 \text{ na } (0,\tfrac{\pi}{2})$$

$$\int \underbrace{\frac{\sqrt{9-9\sin^2 t}}{9\sin^2 t}\,3\cos t\,dt}_{f(\varphi)\cdot\varphi'} =$$



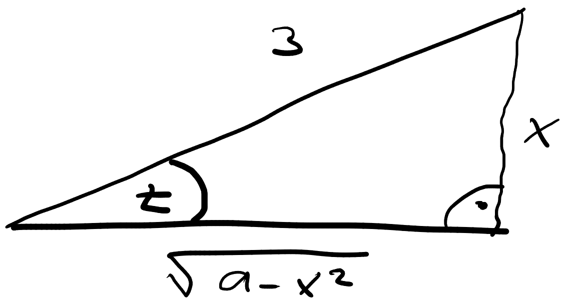


$$= \int \frac{\sqrt{9}\sqrt{1-\sin^2 t}}{3\sin^2 t}\cos t\,dt = \int \frac{\sqrt{\cos^2 t}}{\sin^2 t}\cos t\,dt$$

$$\overset{!}{=} \int \frac{\cos^2 t}{\sin^2 t}\,dt = \int \frac{1-\sin^2 t}{\sin^2 t}\,dt = \int \frac{1}{\sin^2 t} - 1\,dt$$

$$\overset{c}{=} \underbrace{-\cot g\, t - t}_{G(t)} = \underbrace{-\cot g\left(\arcsin\frac{x}{3}\right) - \arcsin\frac{x}{3}}_{G(\varphi^{-1})}$$

$$= -\frac{\sqrt{9-x^2}}{x} - \arcsin\frac{x}{3}$$



$$\frac{x}{3} = \sin t$$

$$\cot t = \frac{\sqrt{9-x^2}}{x}$$

$$\int \cos^2 x \, dx$$

$$\stackrel{c}{=} \frac{1}{2}x + \frac{1}{2} \cdot \frac{\sin 2x}{2}$$

$$\cos^2 x + \sin^2 x = 1$$

$$\cos^2 x - \sin^2 x = \cos 2x$$

$$\cos^2 x = \frac{1 + \cos 2x}{2}$$

- $\cosh^2 x - \sinh^2 x = 1$

- $\int \overbrace{\sqrt{4 + x^2}}^{f(x)} \, dx$ $\quad (a, b) = \mathbb{R}$

$(\alpha, \beta) = \mathbb{R}$

$x = 2 \sinh t$ $\quad \varphi(t) = 2 \sinh t \quad \varphi(\mathbb{R}) \stackrel{na}{=} \mathbb{R}$

$dx = 2 \cosh t \, dt$ $\quad \varphi'(t) = 2 \cosh t$

$\operatorname{argsinh}\left(\frac{x}{2}\right) = t$ $\quad 2 \cosh t \neq 0 \quad t \in \mathbb{R}$ ☺

$$\int \underbrace{\sqrt{4 + 4\sinh^2 t} \quad 2 \cosh t}_{f(\varphi)\,\varphi'} \, dt$$

$$= \int \sqrt{4} \sqrt{1 + \sinh^2 t} \cdot 2 \cosh t \, dt = \int 4 \sqrt{\cosh^2 t} \cdot \cosh t \, dt$$

$$\stackrel{!}{=} \int 4 \cosh^2 t \, dt \stackrel{c}{=} 2 \overbrace{\left(t + \underbrace{\cosh t \cdot \sinh t}_{\frac{1}{2}\sinh(2t)} \right)}^{G(t)}$$

$$= 2 \underbrace{\left(\operatorname{argsinh} \frac{x}{2} + \frac{1}{2} \sinh\left(2 \operatorname{argsinh} \frac{x}{2}\right)\right)}_{G(\varphi^{-1})}$$

$\sqrt{x^2} = |x|$. $\quad \sqrt{x^4} = |x^2| = x^2$

$\sqrt{\cosh^2 t} = |\cosh t| = \cosh t$

$$\int \frac{1}{1 + e^x} \cdot \frac{e^x}{e^x} \, dx$$

$$\int \frac{1}{1 + \sqrt{x}} \cdot \frac{1}{2\sqrt{x}} \cdot 2\sqrt{x} \, dx$$