

Adm. Lösung

$$\int \frac{1 \cdot 3 \sqrt[3]{(x+2)^2}}{\sqrt[3]{(x+2)^2} - 3 \sqrt[3]{x+2} - 4} dx = \frac{1}{3} \frac{1}{\sqrt[3]{(x+2)^2}} dx$$

$$z = \sqrt[3]{x+2} \quad x = z^3 - 2 \quad dx = 3z^2 dz$$

$$\varphi(z) = z^3 - 2$$

$$\varphi' = 3z^2$$

$$\hookrightarrow z \neq 0$$

$$\int \frac{1}{z^2 - 3z - 4} \cdot 3z^2 dz = \int 3 \frac{z^2}{z^2 - 3z - 4} dz = \int 3 \left(-\frac{3/5}{1+z} + \frac{48/5}{z-4} \right) dz$$

$$z \in (-\infty, -1), \quad -1, 4 \in (4, \infty) \Rightarrow (a_1, b_1)$$

$$= 3z - \frac{9}{5} \ln |1+z| + \frac{48}{5} \ln |z-4| =$$

$$= 3 \sqrt[3]{x+2} - \frac{9}{5} \ln |1 + \sqrt[3]{x+2}| + \frac{48}{5} \ln |-4 + \sqrt[3]{x+2}|$$

1 VOS.

$$dz = \frac{1}{3} \frac{1}{\sqrt[3]{(x+2)^2}} dx$$

$$\sqrt[3]{x+2} = z$$

$$z^2 - 3z - 4 \neq 0$$

$$z \neq 4$$

$$(z-4)(z+1) \neq 0$$

$$z \neq -1$$

$$\sqrt[3]{x+2} \neq -1$$

$$x+2 \neq -1$$

$$x \neq -3$$

$$\sqrt[3]{x+2} \neq 4$$

$$x+2 \neq 64$$

$$x \neq 62$$

$$x \in (-\infty, -3), (-3, 62), (62, \infty)$$

$$a_1, b_1$$

$$a_2, b_2$$

$$\varphi = z^3 - 2$$

$$z^3 - 2$$

$$z$$

$$\varphi \text{ NA}$$

$$x$$

$$(a_1, b_1)$$

$$(-\infty, -1)$$

$$\rightarrow (-\infty, -3)$$

$$(a_2, b_2)$$

$$(4, \infty)$$

$$\xrightarrow{\varphi} (62, \infty)$$

$$a_3, b_3$$

$$(-1, 0)$$

$$\xrightarrow{\varphi} (-3, -2)$$

$x_1 \rightarrow x_2$

$$(0, 4) \xrightarrow{\varphi} (-2, 62)$$

$$\underline{\underline{x \in (a, b)}}$$

$$15 - 2 = x \quad \text{le prime}$$

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$$\int \frac{1}{x^2} \sqrt{\frac{1+x}{x}} dx$$

$$\rightarrow x \in (-\infty, -1) \cup (0, \infty)$$

$$(0, 1) \cup (1, \infty)$$

$$t = \sqrt{\frac{1+x}{x}} \quad t^2 = \frac{1+x}{x}$$

$$x t^2 = 1+x$$

$$x(t^2 - 1) = 1$$

$$x = \frac{1}{t^2 - 1}$$

$$dx = \frac{-1}{(t^2 - 1)^2} 2t dt$$

$$t \in \mathbb{R}$$

$$\int \frac{1}{\left(\frac{1}{t^2 - 1}\right)^2} \cdot \frac{1}{t^2 - 1} \cdot \frac{-2t}{(t^2 - 1)^2} dt = \int -2t^2 dt$$

$$\equiv -\frac{2}{3} t^3 = -\frac{2}{3} \sqrt{\left(\frac{1+x}{x}\right)^3}$$

$$x = \varphi(t) = \frac{1}{t^2 - 1}$$

$$\varphi' = \frac{-2t}{(t^2 - 1)^2}$$



$$t \in (\alpha, \beta)$$

$$\xrightarrow{\varphi} (a, b)$$

$$(0, 1)$$

$$\xrightarrow{a_1, b_1} (-\infty, -1)$$

$$(1, \infty)$$

$$\xrightarrow{a_2, b_2} (0, \infty)$$

2V 0 subst. t

$$x \in (-\infty, -1)$$

$$(0, \infty)$$

∴

$$\sqrt{(x-2)(x+3)} = \sqrt{\frac{x-2}{x+3} (x+3)^2}$$

$$= |x+3| \sqrt{\frac{x-2}{x+3}}$$

• $\int \frac{1}{\sqrt{x^2+x+1}} dx \rightarrow x \in \mathbb{R} \text{ a.i.s}$

$$z = \sqrt{x^2+x+1} - x$$

$\Rightarrow \sqrt{x^2}$

$$z+x = \sqrt{x^2+x+1}$$

$$x^2+2x+z^2 = x^2+x+1$$

$$x(2z-1) = 1-z^2$$

$$x = \frac{1-z^2}{2z-1}$$

$$dx = -2 \frac{z^2 - z + 1}{(1-2z)^2}$$

$$\int \frac{1}{z + \frac{1-z^2}{2z-1}} \cdot -2 \cdot \frac{(z^2 - z + 1)}{(1-2z)^2} dz$$

$z \in (-\infty, 1/2) \cup (1/2, \infty)$

$$= \int \frac{-2(z^2 - z + 1)}{2z^2 - z + 1 - z^2} \cdot \frac{1}{(1-2z)^2} dz = \int -2 \frac{-1}{1-2z} dz$$

$+z^2 - z + 1$

$$= \frac{2}{-2} \ln |1-2z| = -\ln |1-2(\sqrt{x^2+x+1} - x)|$$

$x \in (a,b) = \mathbb{R}$

$$\varphi(z) = \frac{1-z^2}{2z-1}$$

$$\varphi' = -2 \cdot \frac{z^2 - z + 1}{(1-2z)^2}$$

$\neq 0$ widdy :)

$\varphi: (\alpha, \beta) \xrightarrow{\varphi} (a, b) = \mathbb{R}$
 $(1/2, \infty) \rightarrow (-\infty, \infty) \checkmark$

~~$(-\infty, 1/2) \rightarrow (-\infty, \infty)$~~

