Určitý ∫ – Newtonův ∫

$$\int_a^b f(x) = \lim_{x\to b-} F(x) - \lim_{x\to a+} F(x)$$

$$\int_0^1 \underbrace{\frac{1}{1+x^2}}_{f(x)} dx = [\operatorname{arctan} x]_0^1 = \operatorname{arctan} 1 - \operatorname{arctan} 0$$

$$= \frac{\pi}{4} - 0 = \underline{\underline{\frac{\pi}{4}}}$$

$$\int_{-1}^{2} 2x\, dx = [x^2]_{-1}^{2} = 2^2 - (-1)^2 = 3$$

$$\int_0^\infty \sin x\, dx = [-\cos x]_0^\infty = \lim_{x\to\infty}(-\cos x) - \lim_{x\to 0+}(-\cos x)$$

! 1. lim $\nexists$ $\longrightarrow$ $\nexists \int_0^\infty \sin x\, dx$

$$\int_0^\infty e^x dx = [e^x]_0^\infty = \lim_{x\to\infty} e^x - \lim_{x\to 0+} e^x = \infty - 1 = \underline{\underline{\infty}}$$

∫ diverguje

$$\int_{-\infty}^{\infty} x\, dx = \left[\frac{x^2}{2}\right]_{-\infty}^{\infty} = \lim_{x\to\infty}\frac{x^2}{2} - \lim_{x\to-\infty}\frac{x^2}{2}$$

∫ $\nexists$

$$= \infty - \infty$$

:(

$$\cancel{\int_{-3}^{3} \frac{1}{x}\, dx \overset{!}{=} \left[\ln|x|\right]_{-3}^{3} = \ln 3 - \ln|-3|} = 0$$

$\frac{1}{x}$ nemá PF na $(-3,3)$

Per partes

$$\int_0^1 \underset{u}{x}\,\underset{v'}{e^x}\, dx = \left[x e^x\right]_0^1 - \int_0^1 e^x\, dx = \left[x e^x\right]_0^1 - \left[e^x\right]_0^1 =$$

$$= e - 0e^0 - (e^1 - e^0) = \underline{1}$$

$u' = 1 \qquad v = e^x$

Substituce

$$\int_0^2 \frac{1}{2\sqrt{x^2+1}} \cdot 2x\, dx = \int_1^5 \underbrace{\frac{1}{2\sqrt{y}}}_{f(y)}\, dy = \left[\sqrt{y}\right]_1^5$$

$$= \sqrt{5} - \sqrt{1} = \sqrt{5} - 1$$

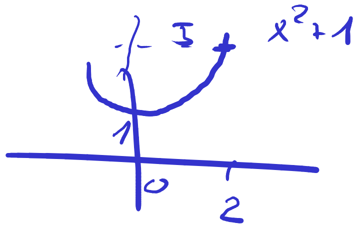
$\varphi(x) \mapsto y = x^2 + 1$

$dy = 2x\, dx$

x	0	2
$y = x^2+1$	1	5

$(\alpha, \beta) = (0, 2)$

$(a, b) = (1, 5)$



f spoj na $[1,5]$? ✓

$\varphi : [0,2] \xrightarrow{\text{do}} [1,5]$? ✓

$\varphi' = 2x$

$2x$ spoj na $[0,2]$? ✓

$$\int_0^{\pi} \sin x \cos x \, dx = \int_0^0 y \, dy = 0$$

$$y = \sin x$$

$$dy = \cos x \, dx$$

x	0	π
y	0	0

$$\int_{-1/2}^{2} \frac{2}{2} \cdot (2x+1) \, dx = \int_0^5 \frac{y}{2} \, dy$$

$$y = 2x+1$$

$$dy = 2 \, dx$$

$$\int \frac{1}{e^x+2}\,\cancel{\frac{e^x}{e^x}\,dx} = \int \underbrace{\frac{1}{(y+2)y}}_{f(y)}\, dy =$$

$\varphi:\ y = e^x$

$dy = \underbrace{e^x}_{\varphi'} dx$

$$= \int \frac{1/2}{y} + \frac{-1/2}{y+2}\, dy =$$

$$\overset{c}{=} \underbrace{\frac{1}{2}\ln|y| - \frac{1}{2}\ln|y+2|}_{F(y)}$$

$$= \underbrace{\frac{1}{2}\ln|e^x| - \frac{1}{2}\ln|e^x+2|}_{F(\varphi(x))}$$

chci: $x \in \mathbb{R}$ $\quad(\alpha,\beta)$

$\varphi(-\infty,\infty) = (0,\infty) \subseteq (a,b)$

$f:\ (\cancel{-\infty,-2})\ (\cancel{-2,0})\ (0,\infty) = (a,b)$

$f(x)$ na $\mathbb{R} = (a,b)$ $\qquad f(\varphi(t))\cdot\varphi'(t)$

$$\int \frac{1}{e^x+2}\,dx = \int \overbrace{\frac{1}{t+2}\cdot\frac{1}{t}}\,dt$$

$e^{\ln t}$ $\qquad (\alpha,\beta)\ (0,\infty)$

$(t = e^x)$ $\qquad \ln t = x \qquad t = e^x$

$\frac{1}{t}\,dt = dx$ $\qquad \varphi(t) = \ln t$

$\varphi^{-1}(x) = e^x$

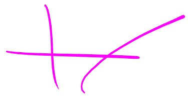
$$\overset{c}{=} \underbrace{\frac{1}{2}\ln|t| - \frac{1}{2}\ln|t+2|}_{G(t)} =$$

$$= \frac{1}{2}\ln|e^x| - \frac{1}{2}\ln|e^x+2| \rightarrow G(\varphi^{-1}(x))$$

$\hookrightarrow \mathbb{R}$

$\varphi(\alpha,\beta) = (a,b)$?

$\ln t \quad (0,\infty) = \mathbb{R}$? ✓

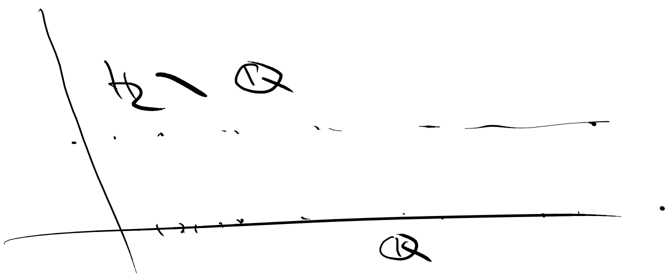


$\varphi' = \dfrac{1}{t} \neq 0$ na $(\alpha,\beta) = (0,\infty)$ ✓

$$\int_0^1 \frac{1}{x}\,dx = \left[\ln|x|\right]_0^1 = 0 - (-\infty) = \infty$$

$$\int_0^1 \frac{1}{x} - \frac{1}{x}\,dx = 0$$

$$\int_0^1 \frac{1}{x}\,dx - \int_0^1 \frac{1}{x}\,dx = \infty - \infty \;\ddot{\frown}$$



$$\int 0 = \int D - \int D$$