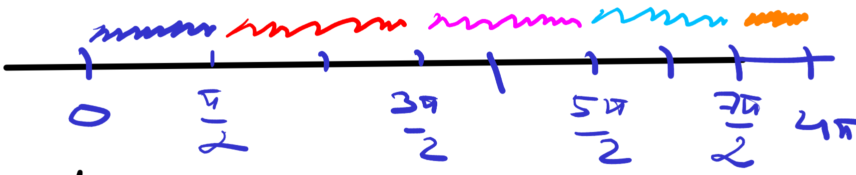
$$\int_0^{4\pi} \frac{1}{\sin^2 x + 2\cos^2 x}\,dx$$



$t = \tan x$ $\qquad dt = \frac{1}{\cos^2 x}\,dx$

$x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ $\quad \left(\frac{\pi}{2}, \frac{3\pi}{2}\right) \dots$

$$\int \frac{1}{\sin^2 x + 2\cos^2 x}\,dx = \int \underbrace{\frac{1}{\frac{t^2}{1+t^2} + 2\frac{1}{1+t^2}}}_{f(t)} \frac{1}{1+t^2}\,dt =$$

$f(\omega(x))\cdot\omega'(x)$

$$= \int \frac{1}{t^2+2}\,dt = \int \frac{1}{2\left(\left(\frac{t}{\sqrt{2}}\right)^2+1\right)}\,dt = \frac{1}{2}\sqrt{2}\arctan\frac{t}{\sqrt{2}} + C$$

$\omega(t) = \tan x$

$$\cancel{\begin{array}{c|c|c} x & 0 & 4\pi \\ \hline t\ \ \tan x & 0 & 0 \end{array}}$$

$$= \cancel{\int_0^0 \frac{1}{t^2+2}\,dt} = 0$$

$$\int_0^{4\pi} = \int_0^{\pi/2} + \int_{\pi/2}^{\frac{3\pi}{2}} + \int_{\frac{3}{2}\pi}^{\frac{5}{2}\pi} + \int_{\frac{5\pi}{2}}^{\frac{7\pi}{2}} + \int_{\frac{7\pi}{2}}^{4\pi}$$

$(\alpha, \beta) = (0, \pi/2)$ $\qquad \omega' = \frac{1}{\cos^2 x} \neq 0$ na (α,β)

$\left(\frac{\pi}{2}, \frac{3\pi}{2}\right), +2\pi, 2k\pi$

$\left(\frac{7\pi}{2}, 4\pi\right)$

(a,b): $\quad \tan((0, \pi/2)) = (0, \infty) = (a,b)$

$\tan\left(\left(\frac{\pi}{2}, \frac{3\pi}{2}\right)\right) = (-\infty, \infty) = (a,b)$

$\tan\left(\left(\frac{7\pi}{2}, 4\pi\right)\right) = (-\infty, 0) = (a,b)$

- $(0, \pi/2)$

$$\rightarrow \int_0^{\infty} = \left[\frac{\sqrt{2}}{2} \operatorname{arctan} \frac{t}{\sqrt{2}}\right]_0^{\infty} = \frac{\sqrt{2}}{2} \cdot \frac{\pi}{2}$$

$(\frac{\pi}{2}, \frac{3\pi}{2})$

$$\rightarrow \int_{-\infty}^{\infty} = \left[\frac{\sqrt{2}}{2} \operatorname{arctan} \frac{t}{\sqrt{2}}\right]_{-\infty}^{\infty} = \frac{\sqrt{2}}{2}\left(\frac{\pi}{2} - -\frac{\pi}{2}\right) = \frac{\sqrt{2}}{2}\pi$$

3 x rovnaký

- $(\frac{7\pi}{2}, 4\pi)$

$$\rightarrow \int_{-\infty}^{0} = \left[\frac{\sqrt{2}}{2} \operatorname{arctan} \frac{t}{\sqrt{2}}\right]_{-\infty}^{0} = -\frac{\sqrt{2}}{2} \frac{-\pi}{2}$$

záver: $\int_0^{4\pi} = \frac{\sqrt{2}}{2}\left(\frac{\pi}{2} + 3\pi + \frac{\pi}{2}\right) = \underline{\underline{2\sqrt{2}\pi}}$