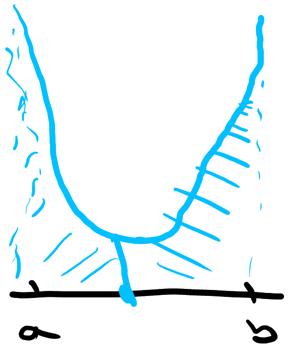
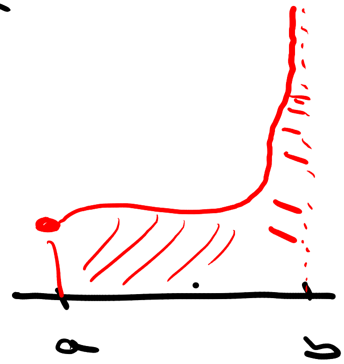
Ak NS

$\int_a^b |f(x)|\, dx$

f spoj. na uz. int.



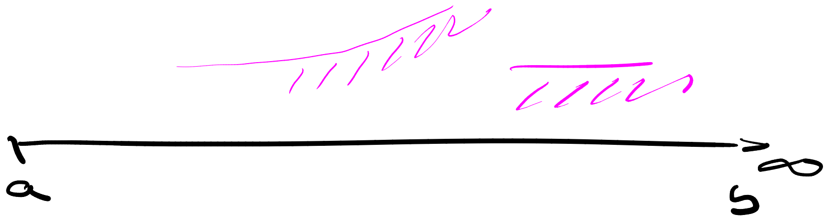


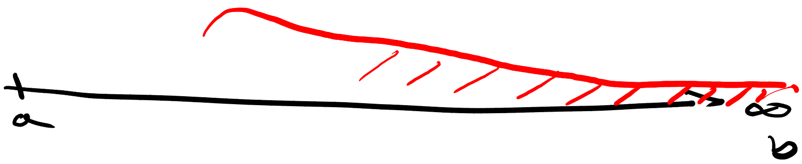
$\frac{1}{\sqrt{x}}$ $\frac{1}{x^2}$

$f \to \infty$

$x \to a+$

$x \to b-$





$\frac{1}{x^2}$:) $\frac{1}{\sqrt{x}}$:(

Př. $$\int_0^{\pi/2} (\sin x)^p (\cos x)^q \, dx$$

• $(a,b) = (0, \pi/2) \to (0, \pi/4) \quad (\pi/4, \pi/2)$

co když $p = -2$

$$\frac{1}{\sin^2 x} \cos x$$

• u 0:

$$\lim_{x \to 0+} \cos x = 1 \quad \checkmark$$

$$\sin x \overset{\text{chová se jako}}{\approx} x$$

$\to \quad g(x) = x^p \cdot 1^q$

LSK

$$\lim_{x \to 0+} \frac{f}{g} = \lim \frac{\sin^p x \cdot \cos^q x}{x^p \cdot 1^q} = \lim \left(\frac{\sin x}{x}\right)^p \cdot \cos^q x$$

$$= 1^p \cdot 1^q = 1 \in (0, \infty)$$

$$\int_0^{\pi/4} x^p \, dx \ K \iff p > -1$$

f, g spoj na $(0, \pi/4]$? Ano.

$g \geq 0$ — u — ?

$$\to \int_0^{\pi/4} f(x) \, dx \ K \iff p > -1$$

u $\frac{\pi}{2}$:

$$\lim_{x \to \frac{\pi}{2}-} \sin x = 1$$

$$\cos x \approx \frac{\pi}{2} - x$$

LSK $\quad g(x) = 1^p \cdot \left(\frac{\pi}{2} - x\right)^q$

$$\lim_{x\to\frac{\pi}{2}-} \frac{f}{g} = \lim_{x\to\frac{\pi}{2}-} \frac{\sin^p x \quad \cos^q x}{1^p \cdot \left(\frac{\pi}{2}-x\right)^q} = \frac{1^p}{1^p}\, 1^q \in (0,\infty)$$

$$\lim_{x\to\frac{\pi}{2}-} \frac{\cos x}{\frac{\pi}{2}-x} \overset{L'H}{\underset{0/0}{=}} \lim_{x\to\frac{\pi}{2}-} \frac{-\sin x}{-1} = 1$$

• $\displaystyle\int_{\pi/4}^{\pi/2} \left(\frac{\pi}{2}-x\right)^q dx = \int_0^{\pi/4} y^q \, dy$ K $\iff q > -1$

$y = \frac{\pi}{2} - x$

$dy = -1\, dx$

x	$\pi/4$	$\pi/2$
y	$\pi/4$	0

• f, g spoj. $\left[\frac{\pi}{4}, \frac{\pi}{2}\right)$ ✓

$g \geq 0$ ✓

$\longrightarrow \displaystyle\int_{\pi/4}^{\pi/2} f$ K $\iff q > -1$

Celkem $\displaystyle\int_0^{\pi/2} f$ K $\iff (p > -1) \& (q > -1)$