Lin rce $\boxed{y' + p(x)\,y = q(x)}$ p, q spoj v int. (a, b)

- $y' - \underbrace{\frac{1}{x+2}}_{p(x)} y = \underbrace{(x+2)}_{q(x)}$ $x \neq -2$

$x \in (-\infty, -2)$, $(-2, \infty)$

- $y' - \frac{1}{x+2} y = 0$

$y' = \frac{1}{x+2} y$ $y \equiv 0$ (řeš.)

$\int \frac{1}{y}\, dy = \int \frac{1}{x+2}\, dx$

$\ln|y| = \ln|x+2| + k$

$|y| = e^k |x+2|$

$y = \underbrace{\pm e^k}_{L} (x+2)$

$y_H = L(x+2)$ $\boxed{L \in \mathbb{R}}$

- $y_p = L(x)(x+2)$

$y_p' = L'(x)(x+2) + L(x)\cdot 1$

$L' \cdot (x+2) + \cancel{L} - \frac{1}{x+2} \cancel{L\cdot(x+2)} = x+2$

$L'(x+2) = (x+2)$, $L' = 1$

$\underline{L = x + D}$ $D \in \mathbb{R}$.

- Závěr: $y = \underline{(x+D)(x+2)}$ $\underline{x \in (-\infty, -2),\ x \in (-2, \infty)}$

- $(x+2)y' - y = (x+2)^2$ $: x+2$

$x \in (-\infty, -2)$ $x \in (-2, \infty)$ $\boxed{x \in \mathbb{R}}$

$y' - \frac{y}{x+2} = x+2$ $\vdots$

$y_1 = (x+D_1)(x+2)$ $x < -2$

$y_2 = (x+D_2)(x+2)$ $x > -2$

Spoj.

y_1 $\quad$ y_2

-2

$$\lim_{x\to -2-} y_1 = \lim_{x\to -2-} (x+D_1)(x+2) = 0 = \lim_{x\to -2+} y_2 = \lim (x+D_2)(x+2)$$

dcc:

$$\lim_{x\to -2-} y_1' = \lim D_1+2x+2 = D_1-2$$

chceme $=$

$$\lim_{y\to -2+} y_2' = \lim D_2+2x+2 = D_2-2$$

$\rightarrow$ $D_1 = D_2$

Závěr

$$y = (x+D_1)(x+2) \qquad x\in\mathbb{R}$$

Splňuje rovnici v $x=-2$

$$(x+2)y' - y = (x+2)^2$$

$x=-2$

$y(-2)=0$

$y' = D_1-2$

$$0(D_1-2) - 0 = 0 \quad \checkmark$$

$$y' - \frac{2}{x+2}y = 2(x+2)\sqrt{y}$$

$y^{1/2}$

$y>0$

$z = y^{1-\alpha}$ $\qquad$ $\alpha = 1/2$

$z = y^{1-1/2} = y^{1/2}$

$z>0$

$y = z^2$ $\qquad$ $y' = 2z\cdot z'$

$z=0$

$$2z\cdot z' - \frac{2}{x+2}z^2 = 2(x+2)z \qquad |:2z$$

$$z' - \frac{z}{x+2} = (x+2) \qquad z>0$$

$$z = (x+2)(x+D)$$

$x\in(-\infty,-2)$ $\quad$ $x\in(-2,\infty)$

$$y = (x+2)^2 (x+D)^2 \longrightarrow \begin{array}{lll} D < 2 & x \in (-D, \infty) & (-\infty, -2) \\ D > 2 & x \in (-2, \infty) & (-\infty, -D) \\ D = 2 & x \in (-2, \infty) & (-\infty, -2) \end{array}$$

$y = 0$ $\quad x \in (-\infty, -2)\ ,\ (-2, \infty)$

v -2 nelze slepit :(

v $-D$ zkusíme slepit, možná to vyjde

ODR n-tého řádu s konst. koef.

- $y''' - 6y'' + 9y' = 0$

λ^n

$\lambda^3 - 6\lambda^2 + 9\lambda^1 = 0$ (λ číslo)

$\lambda(\lambda-3)^2 = 0$

$\lambda_1 = 0$ $\lambda_{2,3} = 3$

2-násob kořen

$y_H = c_1 e^{0x} + c_2 e^{3x} + c_3 x e^{3x}$

$FSŘ = \{1, e^{3x}, xe^{3x}\}$

- $y^{(4)} + 8y'' + 16y = 0$

$\lambda^4 + 8\lambda^2 + 16 \cdot 1 = 0$

$(\lambda^2+4)^2 = 0$

$\lambda^2 + 4 = 0$

$\sqrt{(-1)}\sqrt{(16)}$

$\lambda_{1,2} = \frac{0 \pm \sqrt{0-16}}{2} = \frac{\pm 4i}{2}$

$\lambda_{1,2} = 2i$ $\lambda_3, \lambda_4 = -2i$

$0 + 2i$ $0 - 2i$ (kompl. sdruž.)

2-krát 2-krát

$FSŘ \left\{ e^{0x}\cos 2x,\ e^{0x}\sin 2x,\ xe^{0x}\cos 2x,\ xe^{0x}\sin 2x \right\}$

$$y = c_1 \cos 2x + c_2 \sin 2x + c_3 x \cos 2x + c_4 x \sin 2x$$

$\boxed{x \in \mathbb{R}}$