$$f(x,y) = x^2 - 2xy + 3y^2 + 2 \qquad D_f = \mathbb{R}^2$$

$$\frac{\partial f}{\partial x} = 2x - 2y \qquad D_{\frac{\partial f}{\partial x}}, D_{\frac{\partial f}{\partial y}} = \mathbb{R}^2$$

$$f((2,-1)) = 4 + 4 + 3 + 2 = \cancel{8}\ 13$$

$$\frac{\partial f}{\partial y} = -2x + 6y$$

$$y = kx + q$$

tečná rovina $[a_1, a_2] = [2, -1]$

$$y = f(a) + f'(a)(x - a)$$

$$\frac{\partial f}{\partial x}((2,-1)) = 6$$

$$ax + by + cz + d = 0$$

$$\frac{\partial f}{\partial y}((2,-1)) = -10$$

$$z = f(a_1,a_2) + \frac{\partial f}{\partial x}(a_1,a_2)(x-a_1) + \frac{\partial f}{\partial y}(a_1,a_2)(y-a_2)$$

$$z = 13 + 6(x - 2) + (-10)(y - (-1))$$

tot dif $\qquad f(x,y)$

Lineární zobr. $\quad \mathbb{R}^2 \to \mathbb{R}$

$$(h_1, h_2) \longmapsto \boxed{A h_1 + B h_2}$$

$$A = \frac{\partial f}{\partial x} \qquad B = \frac{\partial f}{\partial y}$$

$$D(f)(2,-1) = \frac{\partial f}{\partial x}(2,-1)\, h_1 + \frac{\partial f}{\partial y}(2,-1)\, h_2$$

$$\boxed{= 6h_1 - 10h_2}$$

$\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}$ spoj.

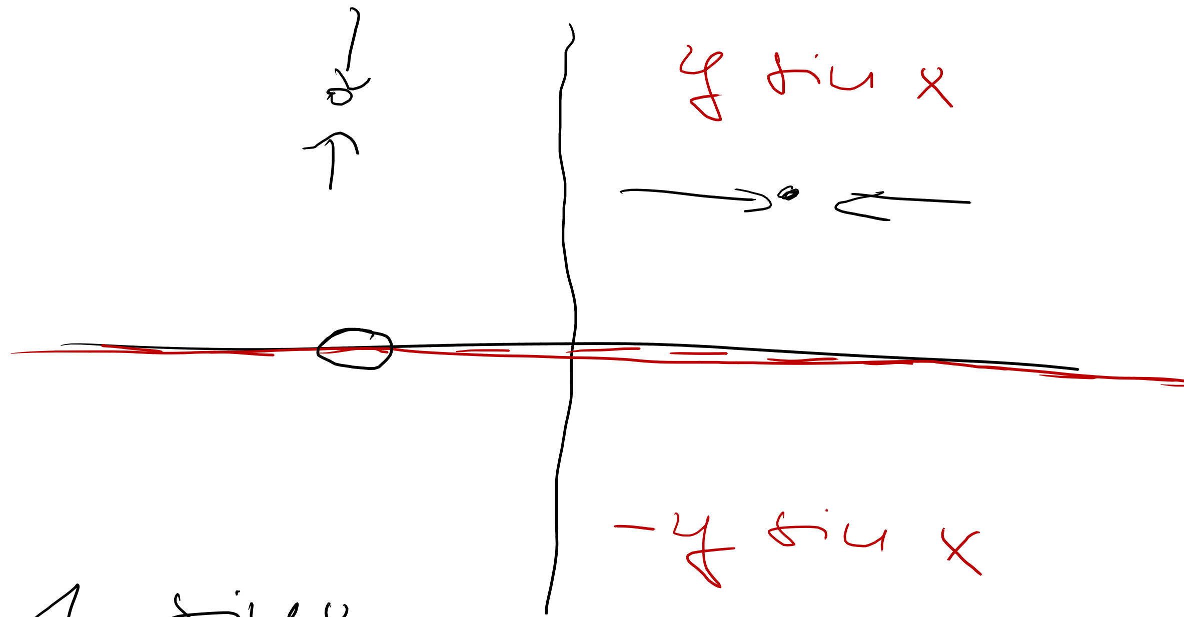
$v\,(2,-1)$

// spoj všude

2a) $f(x,y) = |y| \sin x$ tot. dif $(0,0)$

- $D_f = \mathbb{R}^2$

- $\frac{\partial f}{\partial x} = |y| \cos x$



$\frac{\partial f}{\partial y} = \mathrm{sgn}(y) \cdot 1 \cdot \sin x$

$y \neq 0$

- $y = 0$

$$\frac{\partial f}{\partial y}(x, 0) = \lim_{h \to 0} \frac{f(x, y+h) - f(x,y)}{h}$$

$$= \lim_{h \to 0} \frac{\sin x |y + h| - \sin x |y|}{h} =$$

$$= \lim_{h \to 0} \frac{\sin x |h| - 0}{h} = \sin x \cdot ? \pm 1$$

$$\boxed{x = 0 + k\pi \qquad \lim_{h \to 0} \frac{0}{h} = 0}$$

$x \neq 0 + k\pi$ sgn h

$$\lim_{h \to 0^+} \sin x \frac{\cancel{|h|}\, h}{h} = \sin x$$

$$\lim_{h \to 0^-} = -\sin x$$

v $(0,0)$ $\boxed{\frac{\partial f}{\partial y} = 0}$

$\frac{\partial f}{\partial x} = |y| \cos x$ v $(0,0)$ $\boxed{\frac{\partial f}{\partial x} = 0}$

Tip: $df(0,0) = 0 \cdot h_1 + 0 \cdot h_2$?

$$\lim_{h \to 0} \frac{f(a+h) - f(a) - L(h)}{\|h\|} \overset{?}{=} 0$$

$$\lim_{(h_1,h_2) \to (0,0)} \frac{f(\overset{0,0}{a}+h) - f(a) - (Ah_1 + Bh_2)}{\sqrt{h_1^2 + h_2^2}} \overset{?}{=} 0$$

$f(0+h_1, 0+h_2)$

$$\lim_{h_1,h_2 \to 0,0} \frac{|0+h_2| \cdot \sin(0+h_1) - 0 - (0h_1 + 0h_2)}{\sqrt{h_1^2+h_2^2}}$$

$$= \lim_{h_1,h_2 \to (0,0)} \frac{|h_2| \sin(h_1)}{\sqrt{h_1^2+h_2^2}} = \lim \underbrace{\frac{\sin(h_1)}{h_1}}_{\to 1} \cdot \underbrace{\frac{h_1 |h_2|}{\sqrt{h_1^2+h_2^2}}}_{\to 0}$$

$= \underline{\underline{0}}$ ✓

$|h_1| \leq |h_2|$

$|h_1 h_2| \leq \frac{h_1^2 + h_2^2}{2}$ $| \; | = 0$

$$0 \leq \lim \underbrace{\frac{|h_1||h_2|}{\sqrt{h_1^2+h_2^2}}}_{\text{2 pol} \to 0} \leq \lim \frac{1}{2} \cdot \frac{h_1^2+h_2^2}{\sqrt{h_1^2+h_2^2}} =$$

$$= \lim \frac{1}{2} \sqrt{h_1^2+h_2^2} = 0$$

Závěr: ~~tot~~ dif $(0,0)$

$$\underline{\underline{df(0,0)(h_1,h_2) = 0h_1 + 0h_2 = 0}}$$