① $\lim_{x\to 0} \dfrac{\arctan x - x(1+x)^{1/2} + \frac{x^2}{2}}{x^3} = \lim_{x\to 0} \dfrac{\frac{-5}{24}x^3 + o(x^3)}{x^3} \overset{AL}{=} -\dfrac{5}{24}$

2

1 $\arctan x = x - \frac{x^3}{3} + o(x^4)$

2 $(1+x)^{1/2} = 1 + \frac{1}{2}x + \dfrac{\frac{1}{2}(\frac{1}{2}-1)}{2}x^2 + \dfrac{\frac{1}{2}(\frac{1}{2}-1)(\frac{1}{2}-2)}{3!}x^3 + o(x^3)$ $\quad x\to 0$

4 $f(x) = x - \frac{x^3}{3} + o(x^3) - x\left(1 + \frac{x}{2} - \frac{x^2}{8} + \frac{x^3}{16} + o(x^3)\right) + \frac{x^2}{2}$

$= x^3\left(-\frac{1}{3} + \frac{1}{8}\right) + o(x^3) = -\frac{5}{24}x^3 + o(x^3)$

② $f = \log(1+\sin x) \quad a=0 \quad n=4$

2 $\sin x = x - \frac{x^3}{6} + o(x^4) \quad x\to 0$

2 $\log(1+y) = y - \frac{y^2}{2} + \frac{y^3}{3} - \frac{y^4}{4} + o(y^4) \quad y\to 0$

2 $f(x) = \left(x - \frac{x^3}{6} + o(x^4)\right) - \frac{1}{2}\left(x - \frac{x^3}{6} + o(x^4)\right)^2 + \frac{1}{3}\left(x - \frac{x^3}{6} + o(x^4)\right)^3$

$- \frac{1}{4}\left(x - \frac{x^3}{6} + o(x^4)\right)^4 + o\left(\left(x - \frac{x^3}{6} + o(x^4)\right)^4\right) \quad x\to 0$

3 $= x - \frac{x^3}{6} - \frac{1}{2}\left(x^2 + 2\cdot\frac{-1}{6}x^4\right) + \frac{1}{3}x^3 - \frac{1}{4}x^4 + o(x^4)$

$= x - \frac{x^2}{2} + \frac{x^3}{6} - \frac{x^4}{12} + o(x^4)$

1 $T_n^{f,0} = x - \frac{x^2}{2} + \frac{x^3}{6} - \frac{x^4}{12}$

③ $\sum_{n=1} \left(\dfrac{4n^2+n+2}{5n^2-3n+1}\right)^n$ $\quad 5n^2-3n+1=0 \quad n_{1,2} = \dfrac{3\pm\sqrt{9-20}}{10}$

$a_n \geq 0$, $\quad a_n = |a_n| \to$ K a AK splývají 2

1 Cauchy

2 1

1 $\lim_{n\to\infty} \sqrt[n]{a_n} = \lim_{n\to\infty} \dfrac{4n^2+n+2}{5n^2-3n+1} = \lim_{n\to\infty} \dfrac{4 + \frac{1}{n} + \frac{2}{n}}{5 - \frac{3}{n} + \frac{1}{n}} \overset{AL}{=} \frac{4}{5} < 1$

Tedy dle Cauchyho kritéria řada konverguje. Protože K i AK splývají, tak $\sum a_n$ AK i K.

2

(2)

(1) $\lim_{x\to 0} \frac{\log(1+x) - xe^x + \frac{3}{2}x^2}{x^3} = \lim_{x\to 0} \frac{-\frac{x^3}{6} + o(x^3)}{x^3} \overset{AL}{=} -\frac{1}{6}$

(2) (2)

$\log(1+x) - xe^x + \frac{3}{2}x^2 = x - \frac{x^2}{2} + \frac{x^3}{3} + o(x^3) - x\left(1 + x + \frac{x^2}{2} + \frac{x^3}{6} + o(x^3)\right) + \frac{3}{2}x^2$

$= x^3\left(\frac{1}{3} - \frac{1}{2}\right) + o(x^3) = -\frac{1}{6}x^3 + o(x^3) \quad x \to 0$

(4)

(2) $f = \cos(\sin x) \quad n=4 \quad a=0$

(2) $\sin x = x - \frac{x^3}{6} + o(x^3) \quad x \to 0$

(2) $\cos y = 1 - \frac{y^2}{2} + \frac{y^4}{24} + o(y^5) \quad y \to 0$

(2) $\cos(\sin x) = 1 - \frac{1}{2}\left(x - \frac{x^3}{6}\right)^2 + \frac{1}{24}\left(x - \frac{x^3}{6}\right)^4 + o\left(\left(x - \frac{x^3}{6}\right)^4\right)$

(3) $= 1 - \frac{1}{2}\left(x^2 - \frac{1}{6}\cdot 2\, x^4\right) + \frac{1}{24}x^4 + o(x^4) \quad x \to 0$

$= 1 - \frac{x^2}{2} + \frac{5}{24}x^4 + o(x^4)$

(1) $T_4^{f,0} = 1 - \frac{x^2}{2} + \frac{5}{24}x^4$

(3) $\sum \underbrace{\frac{\sqrt{n+3} - \sqrt{n}}{n}}_{a_n \geq 0} \quad a_n = |a_n| \quad$ Tedy K i AK splývá

(1) $a_n = \frac{n+3-n}{n} \cdot \frac{1}{\sqrt{n+3} + \sqrt{n}} = \frac{3}{n(\sqrt{n+3} + \sqrt{n})}$

AK: LSK s $b_n = \frac{1}{n\sqrt{n}} \quad \sum \frac{1}{n\sqrt{n}}$ K (1)

(2) (2)

$\lim_{n\to\infty} \frac{|a_n|}{b_n} = \lim_{n\to\infty} \frac{\frac{3}{n(\sqrt{n+3}+\sqrt{n})}}{\frac{1}{n\sqrt{n}}} = \lim_{n\to\infty} \frac{3}{\sqrt{1+3/n} + 1} = \frac{3}{2} \in (0,\infty)$

Tedy z LSK $\sum|a_n|$ K $\Leftrightarrow \sum b_n$ K (1)

Protože $\sum b_n$ K, tak i $\sum |a_n|$ K. (1)

K i AK splývají $\rightarrow$ Závěr: $\sum a_n$ AK i K.

(1)

(1)

$$\lim_{x\to 0} \frac{\overbrace{\cos x \sin x - x + \frac{2}{3}x^3}^{f(x)}}{x^5} = \lim_{x\to 0} \frac{\frac{2}{15}x^5 + o(x^5)}{x^5} \overset{AL}{=} \frac{2}{15}$$

(2) (2) (2)

$$f(x) = \left(1 - \frac{x^2}{2} + \frac{x^4}{4!} + o(x^6)\right)\left(x - \frac{x^3}{6} + \frac{x^5}{5!} + o(x^5)\right) - x + \frac{2}{3}x^3$$

$x \to 0$

(4)
$$= x - \frac{x^3}{6} + \frac{x^5}{120} - \frac{x^3}{2} + \frac{x^5}{12} + \frac{x^5}{24} + o(x^5) - x + \frac{2}{3}x^3$$

$$= \frac{2}{15}x^5 + o(x^5)$$

(2) $f(x) = e^{1-\cos x}$ $a=0$ $n=3$

(2) $\cos x = 1 - \frac{x^2}{2} + o(x^3)$ $x \to 0$

$1 - \cos x = \frac{x^2}{2} + o(x^3)$

(2) $e^y = 1 + y + \frac{y^2}{2} + \frac{y^3}{6} + o(y^3)$ $y \to 0$

(3) $e^{1-\cos x} = 1 + \frac{x^2}{2} + o(x^3)$ $1-\cos x \to 0$ $1-\cos x \neq 0$ na $P(0, \pi/4)$

(1) $T_3^{f,0} = 1 + \frac{x^2}{2}$

(3) $\sum \underbrace{(-1)^n \frac{n^2+n+1/n}{\underbrace{n+n\sqrt{n^5+n}}_{a_n \geq 0}}}_{a_n}$

AK: LSK s $b_n = \frac{n^2}{n\sqrt{n^5}} = n^{2-7/2} = n^{-3/2}$ $\sum \frac{1}{n^{3/2}}$ K (1)

(2) (2+1)

$$\lim_{n\to\infty} \frac{|a_n|}{b_n} = \lim \frac{\frac{n^2+n+1/n}{n+n\sqrt{n^5+n}}}{\frac{n^2}{n\sqrt{n^5}}} = \lim \frac{1 + 1/n + 1/n^3}{\frac{1}{\sqrt{n^5}} + \sqrt{1+1/n^4}} \overset{AL}{=} 1 \in (0,\infty)$$

VoS

Tedy $\sum |a_n|$ K $\Leftrightarrow$ $\sum b_n$ K. Protože $\sum b_n$ K, tak i $\sum |a_n|$ K.

(1) (2)

Z AK plyne K.

Závěr: $\sum a_n$ K i AK.

(1)

(A)

1 (1) $\int x\cos(2x) - \frac{\log^2 x}{x}\,dx \overset{c}{=} \frac{x}{2}\sin 2x + \frac{1}{4}\cos 2x - \frac{1}{3}\log^3 x \qquad x \in (0,\infty)$

4 $\int x\cos(2x) \overset{pp}{=} \frac{1}{2}x\sin 2x - \int \frac{1}{2}\sin 2x \overset{c}{=} \frac{x}{2}\sin(2x) + \frac{1}{4}\cos 2x$

$u = x \qquad v' = \cos 2x$

$u' = 1 \qquad v = \frac{1}{2}\sin 2x$

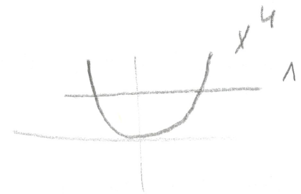
5 $\int \frac{\log^2 x}{x}\,dx \overset{c}{=} \frac{1}{3}\log^3 x$

$(\alpha,\beta) = (0,\infty) \qquad (a,b) = \mathbb{R} \qquad \varphi(\alpha,\beta) \subseteq (a,b)$

φ' ∃ vl. na (α,β)

$y = \log x = \varphi$

$dy = \frac{1}{x}\,dx \;\; \nwarrow \varphi' \qquad \to \int y^2 \overset{c}{=} \frac{1}{3}y^3 \qquad x > 0$



(2) $\int \min\{x^4, 1\}\,dx$

$\to f$ spojité $\mathbb{R}$

1 $f = \begin{cases} 1 & x \in (-\infty,-1] \\ x^4 & x \in (-1,1) \\ 1 & x \in [1,\infty) \end{cases}$

Lemma o lepení 5

2 $F = \begin{cases} x + c_1 & x \in (-\infty,-1) \\ \frac{1}{5}x^5 + c_2 & x \in (-1,1) \\ x + c_3 & x \in (1,\infty) \end{cases}$

$\lim_{x\to -1-} x + c_1 = -1 + c_1$

$\lim_{x\to -1+} \frac{1}{5}x^5 + c_2 = -\frac{1}{5} + c_2$

$\Big\}\; c_1 = \frac{4}{5} + c_2$

2 $F = \begin{cases} x + \frac{4}{5} + c_2 & x \in (-\infty,-1] \\ \frac{x^5}{5} + c_2 & x \in (-1,1) \\ x + c_2 - 4/5 & x \in [1,\infty) \end{cases}$

$\lim_{x\to 1-} \frac{1}{5}x^5 + c_2 = \frac{1}{5} + c_2$

$\lim_{x\to 1+} x + c_3 = 1 + c_3$

$\Big\}\; c_3 = c_2 - \frac{4}{5}$

4

(3) $\int_5^\infty \frac{4}{(x+1)(x-3)}\,dx = \int_5^\infty \frac{\overset{A}{-1}}{x+1} + \frac{\overset{B}{1}}{x-3}\,dx = \Big[-\log|x+1| + \log|x-3|\Big]_5^\infty$ 2

$A(x-3) + B(x+1) = 4$

$x = 3 \qquad B = 1$

$x = -1 \qquad -4A = 4$

$= \Big[\log\frac{|x-3|}{|x+1|}\Big]_5^\infty = \log 1 - \log\frac{2}{6} = \log 3$

1 2 1

(B)

(1) $\int 1 \cdot \arctan(2x)\,dx = \left[x \arctan 2x \right) - \frac{1}{4}\int \frac{2x \cdot 4}{1+(2x)^2}\,dx \overset{c}{=} x \arctan 2x$

$u' = 1 \quad v = \arctan 2x$ (4) (5) $y = 1 + 4x^2$ $\quad -\frac{1}{4}\log(1+4x^2)$

$u = x \quad v' = \frac{2}{1+(2x)^2}$ $\quad dy = 8x\,dx$ $\quad x \in \mathbb{R}$ (1)

$\varphi = 1+4x^2$

$\varphi' = 8x$

$\to -\frac{1}{4}\int \frac{1}{y} = -\frac{1}{4}\log|y|$

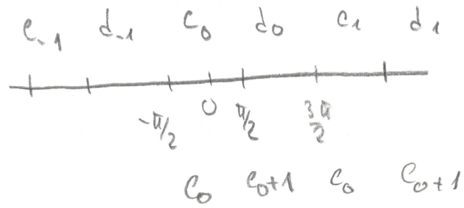
$(\alpha,\beta) = \mathbb{R}$

$\varphi(\alpha,\beta) = [1,\infty)$

$\subseteq (a,b) = (0,\infty)$

(2) $\int \sin x \sqrt{\cos^2 x}\,dx = \int \underbrace{\sin x\,|\cos x|}_{f \text{ spoj na } \mathbb{R}}\,dx$

(2) $f = \begin{cases} \sin x \cos x & x \in (-\frac{\pi}{2}, \frac{\pi}{2}) + 2k\pi \\ -\sin x \cos x & x \in (\frac{\pi}{2}, \frac{3\pi}{2}) + 2k\pi \end{cases}$



(2) (2) $F = \begin{cases} \frac{1}{2}\sin^2 x + c_k \; c_0 & x \in (-\frac{\pi}{2}, \frac{\pi}{2}) + 2k\pi \\ \quad \frac{1}{2} + c_0 & x = \frac{\pi}{2} + k\pi \\ -\frac{1}{2}\sin^2 x + d_k \; c_0+1 & x \in (\frac{\pi}{2}, \frac{3\pi}{2}) + 2k\pi \end{cases}$

(4)

$\lim_{x \to \frac{\pi}{2}- + 2k\pi} F(x) = \frac{1}{2} + c_k$ $\qquad c_k = -1 + d_k$ $\qquad \lim_{x \to -\frac{\pi}{2}+ + 2k\pi} F(x) = \frac{1}{2} + c_k$ $\qquad 1 + c_k = d_k - 1$

$\lim_{x \to \frac{\pi}{2}+ + 2k\pi} F(x) = -\frac{1}{2} + d_k$ $\qquad \lim_{x \to -\frac{\pi}{2}- + 2k\pi} F(x) = -\frac{1}{2} + d_k - 1$

(3) $\int_2^\infty \frac{3}{x^2+x-2}\,dx = \int_2^\infty \frac{3}{(x+2)(x-1)}\,dx = \int_2^\infty \frac{\overset{A}{-1}}{x+2} + \frac{\overset{B}{1}}{x-1}\,dx =$ (4)

$A(x-1) + B(x+2) = 3$

$x = 1 \quad \to B = 1$

$x = -2 \quad -3A = 3$

$= \left[-\log|x+2| + \log|x-1| \right]_2^\infty$ (2) $= \left[\log\left|\frac{x-1}{x+2}\right| \right]_2^\infty$ (1) $= \log 1 - \log \frac{1}{4}$ (2)

$= \log 4$ (1)

(c)

① $\int x^2 \log x - \frac{e^x}{1+e^{2x}}\,dx \stackrel{c}{=} \frac{x^3}{3}\log x - \frac{x^3}{9} - \arctan e^x \qquad x>0$

①

④ $\int x^2 \log x = \frac{x^3}{3}\log x - \int \frac{x^2}{3}\,dx \stackrel{c}{=} \frac{x^3}{3}\log x - \frac{x^3}{9}$

$u' \quad v$

$u = \frac{x^3}{3} \quad v' = \frac{1}{x}$

⑤ $\int \frac{e^x}{1+e^{2x}}\,dx \stackrel{c}{=} \arctan(e^x)$

$y = e^x = \varphi(x) \quad \rightarrow \quad \int \underbrace{\frac{1}{1+y^2}}_{f}\,dy \stackrel{c}{=} \arctan y$

$dy = e^x\,dx$

$\varphi'(x)$

$x \in (-\infty,\infty) = (\alpha,\beta) \qquad (a,b) = \mathbb{R}$

$\varphi(\alpha,\beta) = (0,\infty) \subseteq (a,b)$

② $\int \overbrace{|x(x-4)|}^{f}\,dx$ 0 4

f je spoj. na $\mathbb{R}$ → má na $\mathbb{R}$ PF

1 $f = \begin{cases} x^2-4x & x \in (-\infty,0] \\ -x(x-4) = -x^2+4x & x \in (0,4] \\ x(x-4) = x^2-4x & x \in (4,\infty) \end{cases}$

$\lim_{x\to 0-} F(x) = c_1$

$\lim_{x\to 0+} F(x) = c_2$

$c_1 = c_2$

2 $F = \begin{cases} \frac{x^3}{3} - 2x^2 + c_1 & x \in (-\infty,0) \\ -\frac{x^3}{3} + 2x^2 + c_2 & x \in (0,4) \\ \frac{x^3}{3} - 2x^2 + c_3 & x \in (4,\infty) \end{cases}$

$\lim_{x\to 4-} F(x) = -\frac{64}{3} + 32 + c_2$

$c_3 = -\frac{2\cdot 64}{3} + 64 + c$

$\lim_{x\to 4+} F(x) = \frac{64}{3} - 32 + c_3$

Lemma o lepení

2 $F = \begin{cases} \frac{x^3}{3} - 2x^2 + c_2 & x \in (-\infty,0] \\ -\frac{x^3}{3} + 2x^2 + c_2 & x \in (0,4] \\ \frac{x^3}{3} - 2x^2 + 64 - \frac{128}{3} + c_2 & x \in (4,\infty) \end{cases}$

5

③ $\int_4^\infty \frac{3}{(x+2)(x-1)}\,dx = \int_4^\infty \frac{-1}{x+2} + \frac{1}{x-1}\,dx = \left[-\log|x+2| + \log|x-1|\right]_4^\infty$ ④ ②

$A(x-1) + B(x+2) = 3 \qquad B = 1 \quad A = -1$

$= \left[\log\left|\frac{x-1}{x+2}\right|\right]_4^\infty = 0 - \log\frac{3}{6} = \log 2$

① ② ①

$$\int_0^1 \frac{\sin x}{x^\alpha}\, dx$$

f spoj na $(0,1]$

(2)

(2) LSK s $g = \frac{x}{x^\alpha} = x^{1-\alpha}$ $\quad g$ spoj $\alpha > 0$ na $(0,1]$

(2) $\lim_{x\to 0+} \frac{f}{g} = \lim \frac{\sin x}{x} = 1 \in (0,\infty)$

Tedy $\int_0^1 f$ K $\iff \int_0^1 x^{1-\alpha}$ K $\iff 1-\alpha > -1$ tedy $\alpha < 2$

(2) (2)

Závěr $\int_0^1 f$ K $\iff \alpha < 2$

$$y' = 1+y^2$$

$I = (-\infty,\infty)$

(3) stac. řešení není

$J = (-\infty,\infty)$

(3) $\int \frac{1}{1+y^2}\,dy = \int 1\, dx$

(3) $G(J) = (-\pi/2, \pi/2)$

$\operatorname{arctan} y = x + c$

$-\frac{\pi}{2} < x + c < \frac{\pi}{2}$

(1) $y = \tan(x+c)$ $\quad x \in (-\pi/2 - c, \pi/2 - c)$

$$y' + 2y = x$$

(1) $x \in \mathbb{R}$

$y' = -2y$ $\quad y \equiv 0$

$\int \frac{1}{y}\,dy = \int -2\,dx$

(4) $\log|y| = -2x + c$

$|y| = e^{-2x} e^c$

$y = c e^{-2x}$

$y' = c' e^{-2x} + c e^{-2x}(-2)$

$c' e^{-2x} + c e^{-2x}(-2) + 2 c e^{-2x} = x$

$c' = x e^{2x}$

(4) $\int x e^{2x} = x \frac{1}{2} e^{2x} - \int \frac{1}{2} e^{2x}$

$u \quad v'$

$u' = 1 \quad v = \frac{1}{2} e^{2x}$

$= \frac{x}{2} e^{2x} - \frac{1}{4} e^{2x} + k$

Závěr:

$y = e^{-2x}\left(\frac{x}{2} e^{2x} - \frac{1}{4} e^{2x} + k\right)$

(1) $= \frac{x}{2} - \frac{1}{4} + k e^{-2x}$ $\quad k, x \in \mathbb{R}$

B

(1) $$\int_0^1 \underbrace{\frac{(\arcsin x)^\alpha}{x}}_{f} \, dx$$

(2) $f \geq 0$ na $(0,1]$, spoj. na $(0,1]$

(2) LSK s $g = \frac{x^\alpha}{x}$ $\quad g > 0$ a spoj. na $(0,1]$ $\qquad \int_0^1 x^{\alpha-1}$ K $\iff \alpha - 1 > -1$ $\iff \alpha > 0$

(2) $$\lim_{x\to 0+} \frac{f}{g} = \lim_{x\to 0+} \left(\frac{\arcsin x}{x}\right)^\alpha = 1 \in (0,\infty)$$

(2+2) tedy $\int f$ K $\iff$ $\int g$ K

Závěr: $\int f$ K právě tehdy, když $\alpha > 0$

(2) $$y' = -\frac{x}{y}$$

$I = \mathbb{R}$

(3) stac. řešení není

$J_1 = (-\infty, 0)$, $J_2 = (0, \infty)$

(3) $\int y \, dy = \int -x \, dx$

$\frac{1}{2} y^2 = -\frac{x^2}{2} + c$

$G(y)$

$G(J_1) = G(J_2) = (0, \infty)$

(3) $\to$ $-\frac{x^2}{2} + c > 0$

$2c > x^2$

pro $c \leq 0$ nelze

pro $c > 0$ $\quad \sqrt{2c} > |x|$

$x \in (-\sqrt{2c}, \sqrt{2c})$

$y^2 = -x^2 + 2c$

(1) $y = \pm\sqrt{2c - x^2}$

(3) $$y' + \frac{3y}{x} = \frac{e^x}{x^3}$$

$x \in (-\infty, 0), (0, \infty)$

(1)

$y' = -\frac{3y}{x}$ $\qquad y \equiv 0$

(4) $\int \frac{1}{y} = \int -\frac{3}{x}$

$\log|y| = -3 \log|x| + c$

$|y| = e^{\log \frac{1}{|x|^3}} \cdot e^c$

$y = c \cdot \frac{1}{x^3}$ $\qquad c \in \mathbb{R}$

(4) $y' = c' \cdot \frac{1}{x^3} + c(-3)\frac{1}{x^4}$

$c' \cdot \frac{1}{x^3} + \frac{-3c}{x^4} + \frac{3c}{x^4} = \frac{e^x}{x^3}$

$c' = e^x$

$c = e^x + D$

(1) $y = (e^x + D) \cdot \frac{1}{x^3}$ $\qquad x \in (-\infty, 0)$, $(0, \infty)$

(34)

(1) $\int_0^2 \frac{x^\alpha}{e^x-1}\,dx \quad \alpha\in\mathbb{R}$

(2) f spoj na $(0,2]$, $f \geq 0$ tedy AK a K splývá

(2) LSK s $g = \frac{x^\alpha}{x} = x^{\alpha-1}$ $\quad \int_0^2 x^{\alpha-1}$ K $\iff \alpha-1 > -1 \iff \alpha > 0$ (2)

$g > 0$, spoj na $(0,2]$

(2) $\lim_{x\to 0+} \frac{f}{g} = \lim \frac{x^\alpha}{e^x-1}\cdot\frac{x}{x^\alpha} = 1 \in (0,\infty)$ tedy $\int_0^2 f$ K $\iff \int_0^2 g$ K (1)

(1) Tedy $\int_0^2 f$ K $\iff \alpha > 0$

---

(2) $y' = -\frac{y^2}{x}$ $\quad \cdot\ x \in (-\infty,0),\ (0,\infty)$ (I_1, I_2)

(1) $\cdot\ y_0 \equiv 0$ na J_1, J_2

(2) $\cdot\ y \in (-\infty,0),\ (0,\infty)$ (J_1, J_2)

$G(J_1) = (-\infty,0)$

$G(J_2) = (0,\infty)$ (3)

(3) $\cdot\ \int -\frac{1}{y^2}\,dy = \int \frac{1}{x}\,dx$

$\frac{1}{y} = \log|x| + k$

($\frac{1}{y}$ = $G(y)$)

$|x| < e^{-k}$

$\log|x| < -k$

$\log|x| + k < 0$

$\log|x| + k > 0$

$\log|x| > -k$

$|x| > e^{-k}$

(1) Závěr: $y = \frac{1}{\log|x|+k}$ $\quad k\in\mathbb{R}$ $\quad x \in (-\infty,-e^{-k}),\ (e^{-k},\infty),\ (-e^{-k},0),\ (0,e^{-k})$

(3) $y' + \frac{y}{x} = \frac{1}{x}$ $\quad x \in (-\infty,0),\ (0,\infty)$ (1)

$y' + \frac{y}{x} = 0$

$y' = -\frac{y}{x}$ $\quad y \equiv 0$

$\int \frac{1}{y} = \int -\frac{1}{x}$

(4) $\log|y| = -\log|x| + c$

$|y| = e^{\log\frac{1}{|x|}}\cdot e^{c}$

$y = c\cdot\frac{1}{x}$ $\quad c\in\mathbb{R}$

$y' = c'\cdot\frac{1}{x} + c\cdot\frac{-1}{x^2}$

(4) $c'\cdot\frac{1}{x} - \frac{c}{x^2} + \frac{c}{x^2} = \frac{1}{x}$

$c' = 1$

$c = x + D$

(1) Závěr $y = (x+D)\frac{1}{x}$ na $(-\infty,0),\ (0,\infty)$

$= 1 + \frac{D}{x}$