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1)(a) $\cosh x + \sinh x = e^x$

↳ $\frac{1}{2}(e^x + e^{-x} + e^x - e^{-x}) = \frac{1}{2} \cdot 2 \cdot e^x = e^x$

(b) $\cosh^2 x - \sinh^2 x = 1$

↳ $(\cosh x - \sinh x)(\cosh + \sinh x) =$

$= \frac{1}{2}(e^x + e^{-x} - e^x + e^{-x})(e^x) =$

$= \frac{1}{2} \cdot 2 \cdot e^{-x} \cdot e^x = 1$

(c) $\cosh 2x = \cosh^2 x + \sinh^2 x$

$\cosh 2x = \frac{1}{2}(e^{2x} + e^{-2x})$

$\cosh^2 x + \sinh^2 x = \frac{1}{4}(e^x + e^{-x})^2 + \frac{1}{4}(e^x - e^{-x})^2$

$= \frac{1}{4}(e^{2x} + e^{-2x} + 2e^x e^{-x} + e^{2x} + e^{-2x} - 2e^x e^{-x})$

$= \frac{1}{4}(2e^{2x} + 2e^{-2x})$

(d) $\sinh 2x = 2 \sinh x \cosh x$

$\sinh 2x = \frac{1}{2}(e^{2x} - e^{-2x})$

$2 \sinh x \cosh x = 2 \cdot \frac{1}{2}(e^x - e^{-x}) \frac{1}{2}(e^x + e^{-x})$

$= \frac{1}{2}(e^{2x} - e^{-2x})$

(e) $\cosh(-x) = \frac{1}{2}(e^{-x} + e^{-(-x)}) = \frac{1}{2}(e^x + e^{-x}) = \cosh x$

(f) $\sinh(-x) = \frac{1}{2}(e^{-x} - e^{-(-x)}) = \frac{1}{2}(e^{-x} - e^x) =$

$= -\frac{1}{2}(e^x - e^{-x}) = -\sinh x$

g) $\coth(-x) = \dfrac{\cosh(-x)}{\sinh(-x)} = \dfrac{\cosh(+x)}{-\sinh x} = -\coth x$

h) $\tanh(-x) = \dfrac{\sinh(-x)}{\cosh(-x)} = -\dfrac{\sinh x}{\cosh x} = -\tanh x$

i) $\dfrac{1}{\cos^2 x} = 1 + \tan^2 x$

$$1 + \frac{\sin^2 x}{\cos^2 x} = \frac{\cos^2 x + \sin^2 x}{\cos^2 x} = \frac{1}{\cos^2 x}$$

j) $\dfrac{1}{\cosh^2 x} = 1 - \tanh^2 x$

$$1 - \frac{\sinh^2 x}{\cosh^2 x} = \frac{\cosh^2 x - \sinh^2 x}{\cosh^2 x} = \frac{1}{\cosh^2 x}$$

2a) $\sinh(\ln 3) = \dfrac{e^{\ln 3} - e^{-\ln 3}}{2} = \dfrac{3 - \frac{1}{3}}{2} = \dfrac{4}{3} =$

b) $\cosh(\ln 2) = \dfrac{e^{\ln 2} + e^{-\ln 2}}{2} = \dfrac{2 + \frac{1}{2}}{2} = \dfrac{5}{4} =$

c) $\coth(\ln \frac{1}{3}) = \dfrac{e^{\ln \frac{1}{3}} + e^{-\ln \frac{1}{3}}}{e^{\ln \frac{1}{3}} - e^{-\ln \frac{1}{3}}} = \dfrac{\frac{1}{3} + 3}{\frac{1}{3} - 3} = -\dfrac{5}{4}$

3a) $\sinh x = \frac{3}{4}$

pro to že $\cosh^2 x = 1 + \sinh^2 x$ a $\cosh x > 0$,

máme $\cosh x = \frac{5}{4}$.

Víme, že $\sinh x + \cosh x = e^x$

$e^x = \frac{3}{4} + \frac{5}{4} = 2 \rightarrow \boxed{x = \ln 2}$

3c) $2\cosh 2x + 10 \sinh 2x = 5$

$e^{2x} + e^{-2x} + 5e^{2x} - 5e^{-2x} = 5$

$6e^{2x} - 4e^{-2x} - 5 = 0$ $\quad y = e^{2x}$

$6y - \frac{4}{y} - 5 = 0$

$6y^2 - 4 - 5y = 0$

$(3y - 4)(2y + 1) = 0$

$y = \frac{4}{3}$ $\quad y = -\frac{1}{2}$

$e^{2x} = \frac{4}{3}$ $\quad e^{2x} = -\frac{1}{2}$ nelze

$2x = \log \frac{4}{3}$

$x = \frac{1}{2} \log \frac{4}{3}$

3 (b) $\cosh x = \frac{13}{5}$

$$\frac{1}{2}\left(e^x + e^{-x}\right) = \frac{13}{5}$$

$$y + \frac{1}{y} = \frac{26}{5} \qquad e^x =: y$$

$$y^2 + 1 = \frac{26}{5}y$$

$$y^2 - \frac{26}{5}y + 1 = 0$$

$$y_{1,2} = \frac{\frac{26}{5} \pm \sqrt{\frac{26^2}{25} - 4}}{2}$$

$$= \frac{\frac{26}{5} \pm \sqrt{\frac{676}{25} - \frac{100}{25}}}{2}$$

$$= \frac{\frac{26}{5} \pm \frac{24}{5}}{2} = \begin{cases} 5 \\ \frac{1}{5} \end{cases}$$

$y_1 = 5$

$e^x = 5$

$x = \ln 5$

$y_2 = \frac{1}{5}$

$e^x = \frac{1}{5}$

$y = -\ln 5$

3d) $4\cosh x + \sinh x = 4$

$$4\left(e^x + e^{-x}\right) + e^x - e^{-x} = 4 \cdot 2 \qquad e^x = y$$

$$5e^x + 3e^{-x} = 8$$

$$5y + \frac{3}{y} = 8$$

$$5y^2 - 8y + 3 = 0$$

$$y_{1,2} = \frac{8 \pm \sqrt{64 - 60}}{10}$$

$$y_{1,2} = \frac{8 \pm 2}{10} = \begin{cases} 1 \\ \frac{3}{5} \end{cases}$$

$e^x = 1$

$x = \ln 1$

$x = 0$

$e^x = \frac{3}{5}$

$x = \ln \frac{3}{5}$

$x = \ln 3 - \ln 5$

(4) $\sinh x = \frac{5}{12}$

(a) $\cosh x$ $\qquad \cosh x = \sqrt{1 + \sinh^2 x}$

$\cosh x = \sqrt{1 + \frac{25}{144}}$

$\cosh x = \sqrt{\frac{169}{144}}$

$\cosh x = \frac{13}{12}$

$\cosh x \geq 0 \quad \forall x$

lze v kladku odmocnit

(b) $\coth x = \frac{\cosh x}{\sinh x} = \frac{\frac{13}{12}}{\frac{5}{12}} = \frac{13}{5}$

(c) $\tanh x = \frac{5}{13}$

(d) $\sinh 2x = 2 \sinh x \cosh x = 2 \cdot \frac{5}{12} \cdot \frac{13}{12} = \frac{65}{72}$

(e) $\cosh 2x = \cosh^2 x + \sinh^2 x = \frac{13^2}{12^2} + \frac{5^2}{12^2} = \frac{194}{144} = \frac{97}{72}$

5) $\operatorname{arsinh} x = \ln\left(x + \sqrt{x^2+1}\right)$

$\sinh x = \frac{1}{2}(e^x - e^{-x})$

položme $\quad y = \frac{1}{2}(e^x - e^{-x}) \qquad e^x =: z$

$2y = z - \frac{1}{z}$

$2yz = z^2 - 1$

$0 = z^2 - 2yz - 1$

$z_{1,2} = \frac{2y \pm \sqrt{4y^2+4}}{2}$

$z_{1,2} = y \pm \sqrt{y^2+1}$

$z_2 = y - \sqrt{y^2+1} < 0$

$\Rightarrow$ zahodíme

pak $\quad x = \ln\left(y + \sqrt{y^2+1}\right)$

6) a) $\operatorname{arg\,sinh} \frac{3}{4} = \ln\left(\frac{3}{4} + \sqrt{\frac{9}{16}+1}\right) =$

$\ln\left(\frac{3}{4} + \sqrt{\frac{25}{16}}\right) = \ln 2$

b) $\operatorname{arg\,cosh} 2 = \ln\left(2 + \sqrt{2^2-1}\right) =$

$= \ln(2+\sqrt{3})$

c) $\operatorname{arg\,tanh} \frac{1}{2} = \frac{1}{2}\ln\left(\frac{1+\frac{1}{2}}{1-\frac{1}{2}}\right) = \frac{1}{2}\ln 3$

7) $x = 2\cosh(\vartheta)$

$4\cosh\vartheta \sinh\vartheta = ?$

$\sinh^2\vartheta = \cosh^2\vartheta - 1 = \frac{x^2}{4} - 1 = \frac{x^2-4}{4}$

$|\sinh\vartheta| = \frac{1}{2}\sqrt{x^2-4}$

Pak $4\cosh\vartheta\sinh\vartheta = \operatorname{sgn}(\vartheta) \cdot 4 \cdot \frac{x}{2} \cdot \frac{1}{2}\sqrt{x^2-4}$

↑ znaménko ϑ

Sup + inf

(8)

	inf	min	max	sup
$\mathbb{N}$	1	1	$\nexists$	∞
$(0,2]$	0	$\nexists$	2	2
$(0,1) \cap \mathbb{Q}$	0	$\nexists$	$\nexists$	1
$x \in \mathbb{Z},\ x \geq -\sqrt{6}$	-2	-2	$\nexists$	∞
$(-1)^n \sqrt{n}$	$-\infty$	$\nexists$	$\nexists$	∞
arctan x	$-\frac{\pi}{2}$	$\nexists$	$\nexists$	$\frac{\pi}{2}$
$1 - \frac{1}{n}$	0	0	$\nexists$	1
$\frac{1+(-1)^n}{2}$	0	0	1	1
$\cos \frac{n\pi}{2}$	-1	-1	1	1
$(-1)^n \cdot n$	$-\infty$	$\nexists$	$\nexists$	∞

(9) hapf. $(0,1) \sim \mathbb{R}$

(10) a) $\inf(A \cup B) = \min\{i_A, i_B\}$ $\quad \sup = \max\{s_A, s_B\}$

b) $\inf \geq \max\{i_A, i_B\}$ $\quad \sup \leq \min\{s_A, s_B\}$

c) $\geq i_A$ $\quad \leq s_A$

d) $\geq \min\{i_A, i_B\}$ $\quad \leq \max\{s_A, s_B\}$

e) $= -s_A$ $\quad = -i_A$

f) $= i_A + i_B$ $\quad = s_A + s_B$

g) $= i_A - s_B$ $\quad = s_A - i_B$

h) $\inf = \min\{s_A s_B, s_A i_B, i_A s_B, i_A i_B\}$

$\sup = \max\{$ $-\,''\,-$ $\}$