3rd lesson

`https://www2.karlin.mff.cuni.cz/~kuncova/en/teaching.php`
kuncova@karlin.mff.cuni.cz

Theory

Definition 1. Let $\boldsymbol{A}, \boldsymbol{B} \in M(m \times n)$, $\boldsymbol{A} = (a_{ij})_{\substack{i=1..m\\ j=1..n}}$, $\boldsymbol{B} = (b_{ij})_{\substack{i=1..m\\ j=1..n}}$, $\lambda \in \mathbb{R}$. The *sum of the matrices* $\boldsymbol{A}$ *and* $\boldsymbol{B}$ is the matrix defined by

$$\boldsymbol{A} + \boldsymbol{B} = \begin{pmatrix} a_{11} + b_{11} & a_{12} + b_{12} & \dots & a_{1n} + b_{1n} \\ a_{21} + b_{21} & a_{22} + b_{22} & \dots & a_{2n} + b_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} + b_{m1} & a_{m2} + b_{m1} & \dots & a_{mn} + b_{mn} \end{pmatrix}.$$

The product of the real number λ *and the matrix* $\boldsymbol{A}$ (or the λ-multiple of the matrix $\boldsymbol{A}$) is the matrix defined by

$$\lambda\boldsymbol{A} = \begin{pmatrix} \lambda a_{11} & \lambda a_{12} & \dots & \lambda a_{1n} \\ \lambda a_{21} & \lambda a_{22} & \dots & \lambda a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ \lambda a_{m1} & \lambda a_{m2} & \dots & \lambda a_{mn} \end{pmatrix}.$$

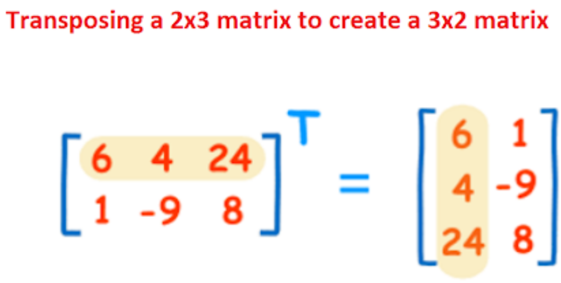
Definition 2. A *transpose* of a matrix

$$\boldsymbol{A} = \begin{pmatrix} a_{11} & a_{12} & a_{13} & \dots & a_{1n} \\ a_{21} & a_{22} & a_{23} & \dots & a_{2n} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & a_{m3} & \dots & a_{mn} \end{pmatrix}$$

is the matrix

$$\boldsymbol{A}^T = \begin{pmatrix} a_{11} & a_{21} & \dots & a_{m1} \\ a_{12} & a_{22} & \dots & a_{m2} \\ a_{13} & a_{23} & \dots & a_{m3} \\ \vdots & \vdots & \ddots & \vdots \\ a_{1n} & a_{2n} & \dots & a_{mn} \end{pmatrix},$$

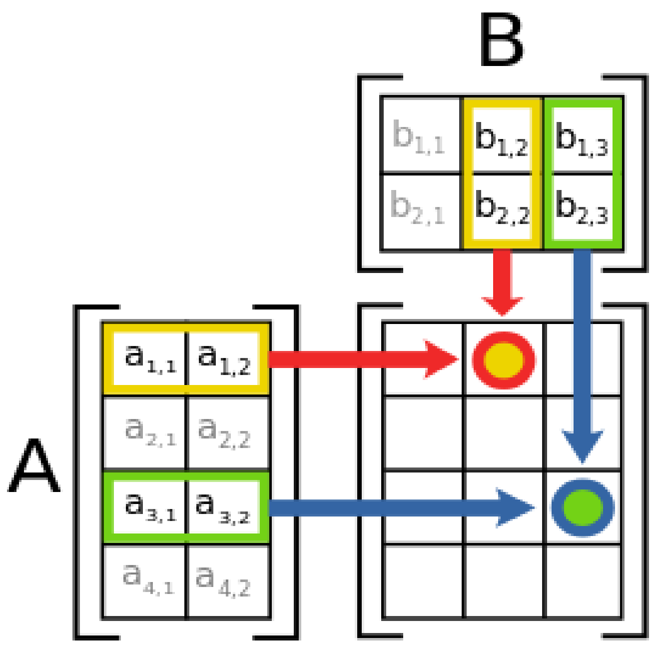
i.e. if $\boldsymbol{A} = (a_{ij})_{\substack{i=1..m\\ j=1..n}}$, then $\boldsymbol{A}^T = (b_{uv})_{\substack{u=1..n\\ v=1..m}}$, where $b_{uv} = a_{vu}$ for each $u \in \{1, \dots, n\}$, $v \in \{1, 2, \dots, m\}$.



Source 1: `https://www.java67.com/2016/10/how-to-transpose-matrix-in-java-example.html`

Definition 3. Let $\boldsymbol{A} \in M(m \times n)$, $\boldsymbol{A} = (a_{is})_{\substack{i=1..m \\ s=1..n}}$, $\boldsymbol{B} \in M(n \times k)$, $\boldsymbol{B} = (b_{sj})_{\substack{s=1..n \\ j=1..k}}$. Then the *product of matrices* $\boldsymbol{A}$ and $\boldsymbol{B}$ is defined as a matrix $\boldsymbol{AB} \in M(m \times k)$, $\boldsymbol{AB} = (c_{ij})_{\substack{i=1..m \\ j=1..k}}$, where

$$c_{ij} = \sum_{s=1}^{n} a_{is} b_{sj}.$$



Source 2: `https://en.wikipedia.org/wiki/Matrix_multiplication`

Exercises

1. Let

$$A = \begin{pmatrix} 6 & -1 \\ 2 & 4 \\ -3 & 5 \end{pmatrix} \qquad B = \begin{pmatrix} 1 & 4 \\ -1 & 5 \\ 1 & 10 \end{pmatrix}$$

Find

(a) $A + B$ (b) $A - B$ (c) $3A$ (d) $3A - 2B$

2. Let

$$A = \begin{pmatrix} -2 & -1 \\ 1 & 0 \\ 3 & -4 \end{pmatrix} \qquad B = \begin{pmatrix} 0 & 3 \\ 2 & 0 \\ -4 & -1 \end{pmatrix}$$

Find X if you know that $2X + B = 2A$

3. Let

$$A = \begin{pmatrix} 2 & 1 \\ 1 & 0 \\ -1 & 2 \end{pmatrix} \qquad C = \begin{pmatrix} 2 & 6 \\ -1 & -3 \end{pmatrix} \qquad E = \begin{pmatrix} 2 & -3 & 0 \end{pmatrix}$$

$$B = \begin{pmatrix} 1 & -1 & 0 & 1 \\ 2 & 1 & 2 & 0 \end{pmatrix} \qquad D = \begin{pmatrix} 1 & 2 \\ 3 & 6 \end{pmatrix} \qquad F = \begin{pmatrix} -5 \\ 2 \\ -2 \end{pmatrix}$$

Find the following products, if they are defined.

(a) AB (c) CD (e) EF (g) A^2

(b) BA (d) DC (f) FE (h) C^2

4. Let

$$A = \begin{pmatrix} 1 & -1 \\ 0 & 2 \end{pmatrix} \qquad B = \begin{pmatrix} 3 & 0 \\ 5 & -1 \end{pmatrix}$$

Find

(a) $(AB)^T$ (b) $A^T B^T$ (c) $B^T A^T$

5. Let

$$A = \begin{pmatrix} a & b & c \\ d & e & f \\ g & h & i \end{pmatrix} \qquad I = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Find

(a) AI (b) IA

6. Let

$$A = \begin{pmatrix} 2 & -1 \\ 1 & 4 \end{pmatrix} \qquad B = \begin{pmatrix} 0 & 0 \\ 3 & -3 \end{pmatrix}$$

Find

(a) AB (b) BA

7. Let

$$A = \begin{pmatrix} 3 & -1 \\ 1 & 3 \end{pmatrix} \qquad B = \begin{pmatrix} 1 & -3 \\ 3 & 1 \end{pmatrix}$$

Find

(a) AB (b) BA

8. True or false? (Assume that all operations are defined.)

(TRUE – FALSE) $A + B = B + A$

(TRUE – FALSE) $AB = BA$

(TRUE – FALSE) $(AB)^T = A^T B^T$

(TRUE – FALSE) $(AB)^T = B^T A^T$

(TRUE – FALSE) $IA = AI = A$

9. Art cinema association- cinemas Oko, Atlas, Aero and Mat - has the following ticket prices. Kids - 40 Kč, Students - 60 Kč and Adults - 80 Kč.

   The attendance of individual cinemas for the last weekend is in the table.

   Use the matrix product to determine the reception of individual cinemas and then determine the reception of the entire association.

	Children	*Students*	*Adults*
Oko	225	110	50
Atlas	75	180	225
Aero	280	85	110
Mat	0	250	225

10. Visit: `https://web.ma.utexas.edu/users/ysulyma/matrix/`

    (You can change the picture: `http://web.natur.cuni.cz/~kunck6am/1819ZS_B1/bilyctverec.jpg`.)

    Try the following matrices:

    (a) i. $\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$ ii. $\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$ iii. $\begin{pmatrix} 0,71 & -0,71 \\ 0,71 & 0,71 \end{pmatrix}$

    (b) i. $\begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$ ii. $\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$ iii. $\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$

    (c) i. $\begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}$ ii. $\begin{pmatrix} 2 & 0 \\ 0 & 1 \end{pmatrix}$ iii. $\begin{pmatrix} 1 & 0 \\ 0 & 2 \end{pmatrix}$

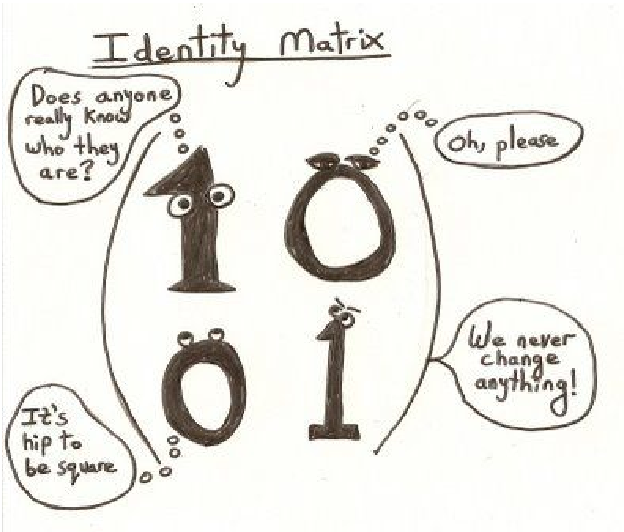
    (d) i. $\begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$ ii. $\begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix}$

11. And what about this matrices? What is the result of matrix multiplying?

    (a) i. $\begin{pmatrix} 0 & 1 \\ -2 & 0 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 2 \end{pmatrix} \cdot \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$

    ii. $\begin{pmatrix} 0 & 2 \\ -1 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 \\ 0 & 2 \end{pmatrix}$

Cheat sheet for matrix transform: `https://en.wikipedia.org/wiki/File:2D_affine_transformation_matrix.svg`



Source 3: https://www.pinterest.com/pin/560698222334789905/